TIME-DEPENDENT ANALYSIS OF PARTIALLY PRESTRESSED CONTINUOUS COMPOSITE BEAMS

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SUMMARY

In this paper a finite element model for the study of creep, shrinkage, and concrete tensile stiffness between cracks effects in partially prestressed continuous composite beams with deformable shear connection is proposed. This model adopts an integral-creep law as constitutive relationship for concrete part under compression. So, step-by-step techniques are used to resolve this integral constitutive relation for any load history. Due to the non-linearity of the problem, the stiffness of the element is calculated with an iterative technique every time-step using a layered model approach for concrete slab.

Keywords: composite, creep, cracking, prestressed, continuous beams, flexible connection.

1. INTRODUCTION

The design of Continuous composite beams for serviceability limit state is not straightforward, since the prediction of behaviour under sustained service loads is complicated by time-dependent deformations in the concrete slab due to creep and shrinkage and additional non-linearity caused by cracking of the concrete over each interior support. The most powerful way that can prevent concrete cracking and improve the stiffness and strength of composite beam is using prestressed concrete. The prestressing can be obtained by relative movements of the internal supports or using internal tendons. Relatively little research has been published on the time-varying behaviour of continuous composite beams subjected to sustained service loads. Dezi and Tarantino in 1993 and 1995 studied the viscoelastic analysis of non-prestressed or fully prestressed composite continuous beams with flexible shear connectors. They consider a nonrealistic assumption that the slab everywhere through the beam is uncracked. Gilbert and Bradford in 1995 described a simple analytical model of the behaviour of non-prestressed cracked continuous composite beams under sustained service loads using the age-adjusted effective modulus. Besides that they neglected the effects of slip between the concrete slab and the steel section as well as the tensile stiffness of concrete between cracks.

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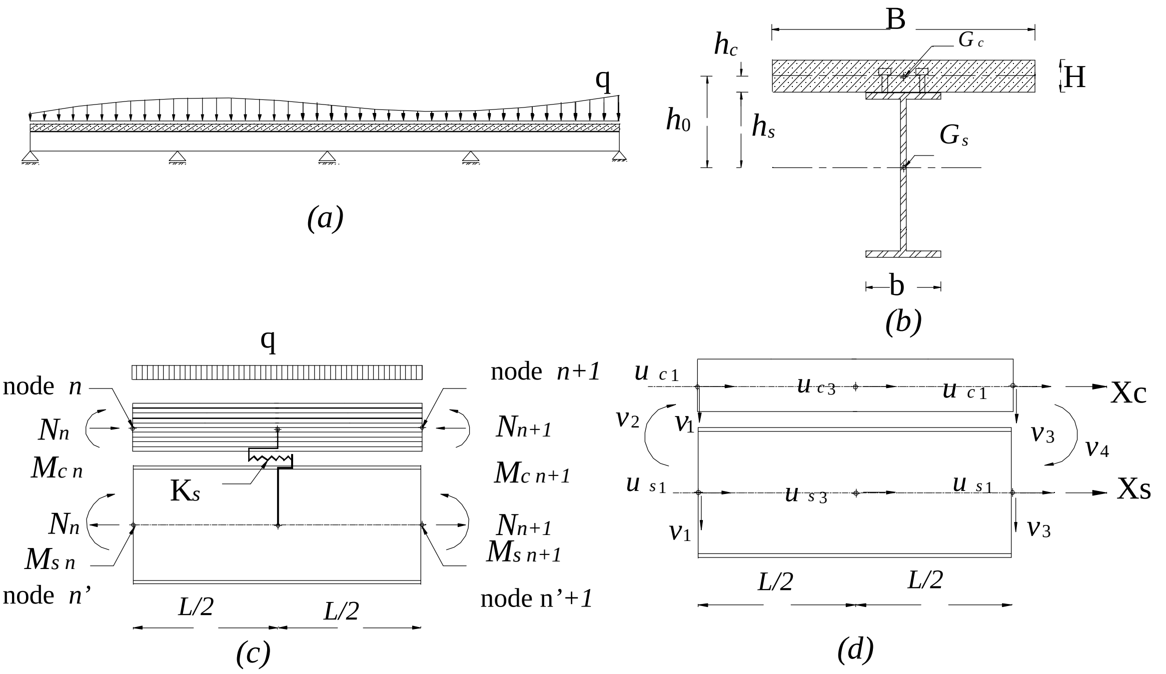

Fig. 1 (a)Continuous composite beam; (b)cross section; (c)an elements connected only at nodes; (d) Nodal displacements of the element(c)[L is the length of the element].

In the present work, the writers intend to develop previous studies by extending the analysis to partially prestressed continuous composite beams. Where the interest in these types of structures is raised because of the wide use of prestressed concrete in the construction of bridges as a way for preventing cracks and developing the durability. This paper describes dependent behaviour of partially prestressed continuous composite beams by a numerical model which is constructed using the finite element method. In this model, a numerical algorithm providing results of good accuracy, has been obtained by employing step-by-step procedures for creep analysis of the general method (Bazant 1972). Incorporation with this, layered model approach is used in order to simplify the complication due to the non-linear behaviour because of cracking over the internal supports. This model makes it possible to evaluate the effects of concrete creep and shrinkage, and stiffness of concrete between cracks on the deformations and stresses of the continuous composite beam at any time. In practical, this model makes it easy to investigate the actual distribution of stress though the composite section at any time for continuous composite beams.

2. FORMULATION OF THE FINITE ELEMENT MODEL

Following the finite element method procedures, the continuous beam has to be replaced by a finite number of elements that are assumed to be connected only at their nodes Fig.1c. The discretization of the beam to specified number of elements has to respect that the shear connectors in the middle of elements. If one considers some elements between the shear connectors the stiffness of connector in these elements will equal zero. The general steps of the finite element method are followed for the derivation of the proposed model. The finite element model proposed by the present study considers the composite beam of Fig.1a as two parallel beam elements with the same deflection v at any cross section linked by means of shear connectors of linear rigidity K_s. Also, this model assumes that no slips between the prestressing steel or the reinforcement steel and the concrete before cracking, and the total prestress losses due to relaxation are concentrated only at the initial time. Finally, the compacted steel sections which have an adequate stiffness against instability are used.

2.1 Constitutive equations of materials

Generally, the concrete slab may contain three different materials (concrete, reinforcement steel, and prestress steel). Because of the a wide difference among the properties of these three materials as well as the difference between the behaviour of concrete in tension and in compression, the layered model approach will be useful for the analysis of such complex element Fig.1c and Fig.2. The hypothesis that concrete can not resist tension is assumed here.

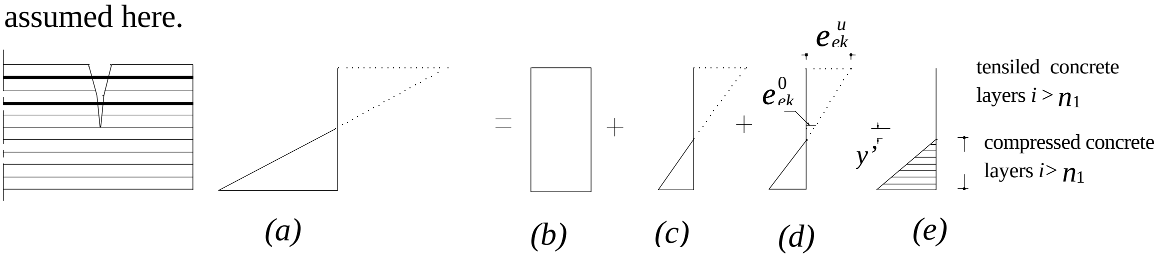

Fig. (2) General layered concrete slab: (a) mean total strain; (b) shrinkage strain; (c) creep strain; (d) elastic strain; and (e) stress distribution.

The concrete layers under compression are assumed to be a linear viscoelastic material governed by the well-known integral-type relationship between a prescribed uniaxial stress history and corresponding strain (Bazant 1972).

$$e_c(t,\ t_0) - e_n(t) = s_c(t_0)\ J(t,\ t_0) + \int_{t_0}^{t} J(t,\ t)\ \mathrm{d}s(t) \tag{2.1}$$

where $e_c(t,\ t_0) =$ the total strain at time t for the applied stress at time t_0; $e_n(t) =$ the shrinkage strain at time t; $s_c(t_0) =$ the stress applied at time t_0; $J(t,\ t_0) =$ the creep or compliance function (strain in time t caused by a constant unite stress applied in time t_0 defined by the CEB Model Code). Solution of structural analysis problems according to the creep law Eq.1 leads to Voltera's integral equations which can be solved only by a step by step numerical method suggested by Bazant 1975 based on the replacement of the Stieltjes heredity integral in Eq.1 by a finite sum. For this purpose time t may be subdivided by discrete times $t_0,\ t_1,\ t_2\ \ldots\ldots,\ t_N$ into subintervals $\Delta t_k = t_k - t_{k-1}$ $k = 1,2,\ldots\ldots N$ which will be considered as unequal. Applying the trapezoidal rule Eq.1 yields

$$\Delta s_{ck} = \overline{E}_c(t_k)\left[\Delta e_{ck} - \Delta e_{vk} - \Delta e_{nk}\right] \tag{2.2}$$

where Δe_{ck}, Δe_{vk}, Δe_{nk} are the incremental of total, viscous(creep), and shrinkage strains respectively; $\Delta e_{vk} = \sum_{I=1..k-1} \frac{\Delta s_{cI}}{2}\left[J(t_k,\ t_I) + J(t_k,\ t_{I-1}) - J(t_{k-1},\ t_I) - J(t_{k-1},\ t_{I-1})\right]$; and $\overline{E}_c(t_k) = 2/\left[J(t_k,\ t_k) + J(t_k,\ t_{k-1})\right]$. Therefore Eq.2 is considered the constitutive relationship of those layers under sustained compressive stress($y_i \le y_k'$). y_i is y co-ordinate of the layer number *i* (*i*=1,2,3..... n (number of concrete layers)). y_k' is the co-ordinate at which e_e equals zero; in time t_k, $y_k' = -e_{ek}^{0}/a_k$, $a_k = e_{ek}^{u} - e_{ek}^{0}/H/2$.

When a crack is formed, the stress in the concrete immediately adjacent to the crack drops to zero and the steel stress increases to that corresponding to the fully cracked state. With increasing distance from the crack the stress in the concrete increases as force is transferred from the steel to the concrete by bond stress, t_b, until at some distance, l_t, from the crack the concrete again attains the tensile strength the concrete Fig.3. It should be noted that crack formation is an inherently semi-random process. The empirical formula suggested by CEB[1967-1978], for crack spacing, l (in mm), that will be recommended here is:

$$l = 2(c + s/10) + (k_2\, f_{ct}/t_b)(A_{ct}/u_s) \qquad (2.3)$$

where c = the concrete cover (in mm); s = spacing of bars (in mm); A_{ct} = area of concrete in tension; u_s = the sum of the perimeters of the reinforcing bars; and k_2 = a coefficient allowing for the effect of the distribution of tensile stress within the section, equals 1.0 for pure tension and 0.5 for pure bending.

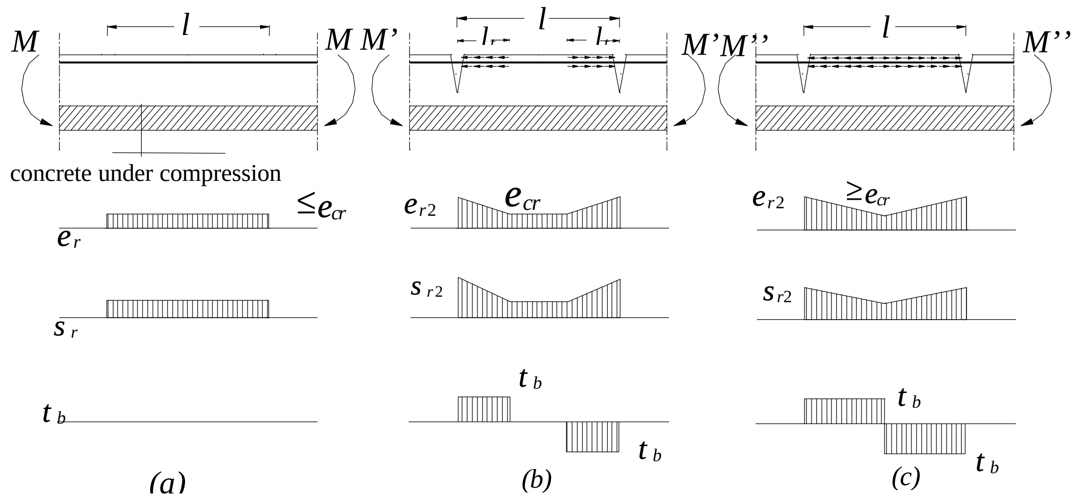

Fig 3 The different stages of the strain and stress Distributions between the cracks for reinforcement in partially cracked concrete slab. [M < M '< M'']

It is assumed that the bond stress is independent of the displacement of the steel bar relative to the outer surrounding concrete. So, the distance l_t is calculated as follows:

$$l_t = A_s\, E_r (e_{r2} - e_{cr}) / (t_b u_s) \qquad (2.4)$$

where $e_{cr} = f_{ct}/\overline{E}_c(t)$; e_{r2} = the reinforcement strain at the crack position; and E_r = reinforcement steel Young's modulus. With regard to the stress of the reinforcement at the crack position three stages can be distinguished see Fig 3. The incremental form of the stress at the crack position Δs_{r2k} - mean strain Δe_{rm} relationship can be written as:

$$\Delta s_{r2k} = E_r [\Delta e_{rmk} + ¶\, e_{rk}] \qquad (2.5)$$

where $¶\, e_{rsk} = \Delta e_r(t_k) - \Delta e_r(t_{k-1}), \Delta e_r(t_{k-1})$ is memorised through the calculation, $\Delta e_r(t_k) = 0.0$ for $l_{tk} \leq 0.0$, $\Delta e_r(t_k) = (1 - l_t/L)[e_{r2} - e_{cr}]$ for $0.0 < l_{tk} < l_k / 2$, $\Delta e_r(t_k) = [t_b u_s L]/[4 E_s A_s]$ for $l_{tk} \geq l_k / 2$; and l_{tk}, l_k are the values of l_t, l in time t_k. In the same way, the behaviour of prestressed steel in concrete slab of composite continuous may be governed by Eq. 5. It should be noted that the value of $\P e_{pk}$ must be calculated with the bond stress between the prestressed steel and the surrounding concrete. The current study considers the connection between the concrete slab and steel section only at the shear connector's position. With the assumption that the concrete slab and the steel beam deform with the same curvature, one gets the following relation:

$$g = h_0 [dv/dx]_{x=x_n} - (u_{cn} - u_{sn}) \tag{2.6}$$

where g = the slip between the concrete slab and steel section at the shear connector's location $x = x_n$; $[dv/dx]_{x=x_n}$ = the rotation of the composite section at the shear connector's location; and u_{cn}, u_{sn} = the horizontal displacements of the points on the middle surfaces of the concrete slab and the steel beam at the shear connector's location, respectively. The shear force resisted by the connector Q can be estimated in an incremental form as:

$$\Delta Q_k = K_s \Delta g_k \tag{2.7}$$

The stress-strain relationship of steel section can be assumed linear where the serviceability stresses are fare less than the ultimate strength.

2.2 Element formulation

With reference to the parameters of the nodal displacements of the element proposed in Fig.1d and according to usual technique for finite elements it is possible to suppose:

$$\Delta u_{sk}(x) = \mathbf{N}_s \Delta U_{sk} \qquad \Delta u_{ck}(x) = \mathbf{N}_c \Delta U_{ck} \qquad \Delta v_k(x) = \mathbf{N}_v \Delta \mathbf{V}_k \tag{2.8}$$

where $\Delta U_c^T = \{\Delta u_{c1}, \Delta u_{c2}, \Delta u_{c3}\}$; $\mathbf{N}_c = \{N_{c1}(x), N_{c2}(x), N_{c3}(x)\}$; and analogously for the vectors ΔU_s^T, $\Delta \mathbf{V}_k$ and for matrices of shape functions $\mathbf{N}_s$, $\mathbf{N}_v$. The incremental horizontal displacement of the shear connector can be written as:

$$\Delta g_k = \mathbf{A}_1 \Delta U_{ck} + \mathbf{A}_2 \Delta U_{sk} + h_0 \mathbf{N}'_{vg} \Delta \mathbf{V}_k \tag{2.9}$$

where $\mathbf{A}_1 = \{0, 0, -1\}$; $\mathbf{A}_2 = \{0, 0, 1\}$; and $\mathbf{N}'_{vg}$ = the matrix of first derivative of $\mathbf{N}_v$ at x = L/2. The incremental mean strains $\Delta e_{cmk}(x)$ of the concrete layer(i) can be expressed as a function of incremental nodal displacements

$$\Delta e_{cmk}(x) = \mathbf{N}'_c \Delta U_{ck} - y_i \mathbf{N}''_v \Delta \mathbf{V}_k \tag{2.10}$$

where $\mathbf{N}'_c$ = the first derivatives of the matrices $\mathbf{N}_c$; $\mathbf{N}''_v$ = the second derivatives of the matrix $\mathbf{N}_v$, and y_i = the distance from the middle of concrete slab to the centroid of the concrete layer. Similarly, one gets the incremental mean strains of reinforcement steel and prestressed steel, and steel section. Applying the principle of virtual work to a generic instant t_k with one element, considering the creep, shrinkage, relaxation, change of element stiffness, and load applied in the time interval (t_{k-1}, t_k) for the iteration number j one gets

$$\int_{V_c} d(\Delta e_{cmk}) \Delta s_{ckj} dV_c + \int_{V_s} d(\Delta e_{smk}) \Delta s_{skj} dV_s + d(\Delta g_k) \Delta Q_{kj} = \int_L d(\Delta v_k) \Delta q_k dx \tag{2.11}$$

where V_c, V_s represent the volume of concrete and steel respectively, Δq_k the possible increment of distributed load applied to the element in the time interval $(t_{k-1},\ t_k)$. Considering the constitutive relations and the strain-displacement relations in the previous section incorporated with the layered system assumed for concrete slab Fig.1, one obtains:

$$\sum_{i=1}^{n1kj} \mathrm{B} h_i \int_L \left[d\left(\Delta U_{ck}^T\right) \mathbf{N}'_c{}^T - y_i d\left(\Delta \mathbf{V}_k{}^T\right) \mathbf{N}''_v{}^T \right] \overline{E}_c(t_k) \left[\mathbf{N}'_c\ \Delta U_{ck}^j - y_{ic} \mathbf{N}''_v\ \Delta \mathbf{V}_k^j - \Delta e_{vk} - \Delta e_{nk} \right] dx +$$
$$\sum_{i=n1kj}^{n} B h_i \int_L \left[d\left(\Delta U_{ck}{}^T\right) \mathbf{N}'_c{}^T - y_i d\left(\Delta \mathbf{V}_k{}^T\right) \mathbf{N}''_v{}^T \right] \overline{E}_c(t_k) \left[-\Delta e_{nk} \right] dx +$$
$$B h_r \int_L \left[d\left(\Delta U_{ck}{}^T\right) \mathbf{N}'_c{}^T - y_r d\left(\Delta \mathbf{V}_k{}^T\right) \mathbf{N}''_v{}^T \right] E_r \left[\mathbf{N}'_c \Delta U_{ck}^j - y_r \mathbf{N}''_v\ \Delta \mathbf{V}_k^j + \P^j e_{rk} \right] dx +$$
$$B h_p \int_L \left[d\left(U_{ck}^T\right) \mathbf{N}'_c{}^T - y_p d\left(\Delta \mathbf{V}_k{}^T\right) \mathbf{N}''_v{}^T \right] E_p \left[\mathbf{N}'_c \Delta U_{ck}^j - y_p \mathbf{N}''_v\ \Delta \mathbf{V}_k^j + \P^j e_{pk} \right] dx +$$
$$\int_{Vs} \left[d\left(\Delta U_{sk}\right) \mathbf{N}'_s{}^T - y_i d\left(\Delta \mathbf{V}_k\right) \mathbf{N}''_v{}^T \right] E_s \left[\mathbf{N}'_s \Delta U_{sk}^j - y_{si} \mathbf{N}''_v \Delta \mathbf{V}_k^j \right] dx +$$
$$\left[d\left(\Delta U_{ck}{}^T\right) \mathbf{A}_1{}^T + d\left(\Delta U_{sk}{}^T\right) \mathbf{A}_2{}^T + h_0 d\left(\Delta \mathbf{V}_k{}^T\right) \mathbf{N}'_{vg}{}^T \right] K_s \left[\mathbf{A}_1 \Delta U_{ck}^j + \mathbf{A}_2 \Delta U_{sk}^j + h_0 \mathbf{N}'_{vg}\ \Delta \mathbf{V}_k^j \right]$$
$$= \int_L d\left(\Delta \mathbf{V}_k^T\right) \mathbf{N}_v^T \Delta q_x\, dx \qquad (2.12)$$

where B = the width of the concrete slab; h_i, h_r, h_p are the thickness of the concrete, reinforcement steel, prestressing steel layers; E_s, E_p are steel and prestressed steel Young's modulus and n_{1kj} is the numbers of concrete layers under compression. As the equation of the virtual work Eq.12 must be respected for every congruent $d\left(\Delta U_{ck}{}^T\right)$, $d\left(\Delta U_{sk}{}^T\right)$, $d\left(\Delta V_k{}^T\right)$ the system of three equations immediately can be obtained:

$$\left[EAc_{ek}^j \int_L \mathbf{N}'_c{}^T \mathbf{N}'_c\, dx + K_s \mathbf{A}_1{}^T \mathbf{A}_1 \right] \Delta U_{ck}^j + \left[K_s \mathbf{A}_1{}^T \mathbf{A}_2 \right] \Delta U_{sk}^j +$$
$$\left[ESc_{ek}^j \int_L \mathbf{N}'_c{}^T \mathbf{N}''_v\, dx + K_s h_0 \mathbf{A}_1{}^T \right] \Delta \mathbf{V}_k^j = EAc_{1k} \int_L \mathbf{N}'_c{}^T \Delta e_{vk}\, dx + EAc_k \int_L \mathbf{N}'_c{}^T \Delta e_{nk}\, dx +$$
$$+ EA_r \int_L \mathbf{N}'_c{}^T \P^j e_{rk}\, dx + EA_p \int_L \mathbf{N}'_c{}^T \P^j e_{pk}\, dx \qquad (2.13)$$

$$\left[K_s \mathbf{A}_2{}^T \mathbf{A}_1 \right] \Delta U_{ck}^j + \left[E_s A_s \int_L \mathbf{N}'_s{}^T \mathbf{N}'_s\, dx + K_s \mathbf{A}_2{}^T \mathbf{A}_2 \right] \Delta U_{sk}^j + \left[K_s h_0 \mathbf{A}_2{}^T \right] \Delta \mathbf{V}_k^j = 0.0 \qquad (2.14)$$

$$\left[-ESc_{ek}^j \int_L \mathbf{N}''_v{}^T \mathbf{N}'_c\, dx + K_s h_0 \mathbf{N}'_{vg}{}^T \mathbf{A}_1 \right] \Delta U_{ck}^j + \left[K_s h_0 \mathbf{N}'_{vg}{}^T \mathbf{A}_2 \right] \Delta U_{ck}^j$$
$$+ \left[EI_{ek}^j \int_L \mathbf{N}''_v{}^T \mathbf{N}''_v\, dx + K_s h_0^2 \mathbf{N}'_{vg}{}^T \mathbf{N}'_{vg} \right] \Delta \mathbf{V}_k^j = \int_L \mathbf{N}_v{}^T \Delta q_x\, dx + ESc_{1k} \int_L \mathbf{N}''_v{}^T \Delta e_{vk}\, dx +$$
$$ES_r \int_L \mathbf{N}''_v{}^T \P^j e_{rk}\, dx + ES_p \int_L \mathbf{N}''_v{}^T \P^j e_{pk}\, dx \qquad (2.15)$$

where $EAc_{1k} = \sum_{i=1}^{n1(k-1)} B h_i \overline{E}_c(t_k)$; $\quad ESc_{1k} = \sum_{i=1}^{n1(k-1)} \mathrm{B} h_i y_{ci} \overline{E}_c(t_k)$; $\quad EAc_k = \sum_{i=1}^{n} \mathrm{B} h_i \overline{E}_c(t_k)$;

$EA_r = E_r B h_r$; $\quad ES_r = E_r B h_r y_r$; $\quad EA_p = E_p B h_p$,

$ES_p = E_p A_p y_p$; $\quad EAc_{ek}^j = \sum_{i=1}^{n1kj} \mathrm{B} h_i \overline{E}_c(t_k), + EA_r + EA_p$,

$$ESc^{j}_{ek} = \sum_{i=1}^{n1kj} B\, h_i\, y_{ci}\, \overline{E}_c(t_k), + ES_r + ES_p\text{, and}$$

$$EI^{j}_{ek} = \sum_{i=1}^{n1kj} B\, h_i\, y^{2}_{ci}\, \overline{E}_c(t_k) + E_r\, B\, h_r\, y^{2}_{r} + E_{pp}\, B\, h_p\, y^{2}_{p} + I_s\, E_s .$$

Eqs. 11, 12 and 13 can put in the following form:

$$\mathbf{K}_{11}{}^{j}_{k}\, \Delta U^{j}_{ck} + \mathbf{K}_{12}{}^{j}_{k}\, \Delta U^{j}_{sk} + \mathbf{K}_{13}{}^{j}_{k}\, \Delta \mathbf{V}^{j}_{k} = \Delta \mathbf{f}^{v}_{1k} + \Delta \mathbf{f}^{n}_{1k} + \Delta \mathbf{f}^{jr}_{1k} + \Delta \mathbf{f}^{jp}_{1k} \tag{2.16}$$

$$\mathbf{K}_{21}{}^{j}_{k}\, \Delta U^{j}_{ck} + \mathbf{K}_{22}{}^{j}_{k}\, \Delta U^{j}_{sk} + \mathbf{K}_{23}{}^{j}_{k}\, \Delta \mathbf{V}^{j}_{k} = 0.0 \tag{2.17}$$

$$\mathbf{K}_{31}{}^{j}_{k}\, \Delta U^{j}_{ck} + \mathbf{K}_{32}{}^{j}_{k}\, \Delta U^{j}_{sk} + \mathbf{K}_{33}{}^{j}_{k}\, \Delta \mathbf{V}^{j}_{k} = \Delta \mathbf{f}^{v}_{3k} + \Delta \mathbf{f}^{q}_{3k}\, \Delta \mathbf{f}^{jr}_{3k} + \Delta \mathbf{f}^{jp}_{3k} \tag{2.18}$$

$\mathbf{K}_{11}{}^{j}_{k}$, $\mathbf{K}_{12}{}^{j}_{k}$,... are sub-matrices. One can rearrange the equations and carry out a static condensation procedure of the equations of the internal nodal parameters u_{c3} and u_{s3} to obtain a reduced system of the type:

$$\underline{\mathbf{K}}^{ej}_{k(8x8)}\, \Delta \underline{U}^{ej}_{k(1x8)} = \Delta \underline{\mathbf{f}}^{ej}_{k(1x8)} \tag{2.19}$$

where the matrices $\underline{\mathbf{K}}^{ej}_{k}$ = condensed stiffness matrix of the element, and $\Delta \underline{U}^{ejT}_{k} = \{\Delta u^{j}_{c1k},\ \Delta u^{j}_{s1k},\ \Delta v^{j}_{1k},\ \Delta v^{j}_{2k},\ \Delta u^{j}_{c2k},\ \Delta u^{j}_{s2k},\ \Delta v^{j}_{3k},\ \Delta v^{j}_{4k}\}$. The values of the load sub-vectors $\Delta \mathbf{f}^{n}_{1k}$ and $\Delta \mathbf{f}^{q}_{3k}$ can be easy obtained by integrating the shape functions

$$\Delta \mathbf{f}^{nr}_{1k} = EAc_k \int_L \mathbf{N'}_c{}^T \Delta e_{nk}\, dx = EAc_k\, \Delta e_{nk}\, \mathbf{Z}_1 \qquad \Delta \mathbf{f}^{q}_{3k} = \int_L \mathbf{N}_v{}^T \Delta q_x\, dx = \Delta q_x\, \mathbf{Z}_2 \tag{2.20}$$

where $\mathbf{Z}^{T}_{1} = \{-1,\ 1,\ 0.0\}$ and $\mathbf{Z}^{T}_{2} = \{L/2,\ L^2/12,\ L/2,\ -L^2/12\}$.

Four values of the concrete elastic strains for any element will be calculated and memorised at the end of each time increment. These values are the elastic strains in the middle $(\Delta e^{0}_{e1k}, \Delta e^{0}_{e2k})$ and upper $(\Delta e^{u}_{e1k}, \Delta e^{u}_{e2k})$ fibres of concrete slab at nodes 1 and 2 of the element considering the creep for the whole section. These strains are calculated using the fact that the total strain can be divided to elastic, shrinkage and creep strains. Using these four values the stains at nodes 1 and 2 for any layer, the elastic strain for any layer which is a linear function through the element can be estimated. Then, the load sub-vector $\Delta \mathbf{f}^{v}_{1k}$ can be written in the following form:

$$\Delta \mathbf{f}^{v}_{1k} = EAc_{1k} \int_L \mathbf{N'}_c{}^T \Delta e_{vk}\, dx = \mathbf{K}^{*} \sum_{I=1}^{k-1} R(t_k, t_I)\, \overline{E}_c(t_I) \left[EAc_{1k}\, \Pi_{1I} + ESc_{1k}\, \mathrm{A}_{1I} \right] \tag{2.21}$$

where $\mathbf{K}^{*} = 0.5 \int_L \mathbf{N'}_c{}^T \mathbf{B}_1\, dx$; $\mathbf{B}_1 = \{(L-x)/L,\ x/L,\ 0.0\}$; $\Pi^{T}_{1k} = \{e^{0}_{e1k},\ e^{0}_{e2k},\ 1\}$, and $\mathrm{A}^{T}_{1k} = \{(\Delta e^{u}_{e1k} - \Delta e^{0}_{e1k})/(H/2),\ (\Delta e^{u}_{e2k} - \Delta e^{0}_{e2k})/(H/2),\ 1\}$. For the sub-vector $\Delta \mathbf{f}^{v}_{3k}$, in an analogous way one obtains:

$$\Delta \mathbf{f}^{v}_{3k} = ESc_{1k} \int_L \mathbf{N''}_v{}^T \Delta e_{vk}\, dx = \mathbf{K}^{**} \sum_{I=1}^{k-1} R(t_k, t_I)\, \overline{E}_c(t_I) \left[ESc_{1k}\, \Pi_{3I} + EIc_{1k}\, \mathrm{A}_{3I} \right] \tag{2.22}$$

where $\mathbf{K}^{**} = 0.5 \int_L \mathbf{N''}_v{}^T \mathbf{B}_3\, dx$, $EIc_{1k} = \sum_{i=1}^{n1(k-1)} B\, h_i\, y^{2}_{ci}\, \overline{E}_c(t_k)$, $\Pi^{T}_{1k} = \{e^{0}_{e1k},\ e^{0}_{e2k},\ 1,\ 1\}$,

$\mathbf{B}_3 = \{(L-x)/L,\ x/L,\ 0.0,\ 0.0\}$, and $\mathrm{A}^{T}_{1k} = \{(\Delta e^{u}_{e1k} - \Delta e^{0}_{e1k})/(H/2),\ (\Delta e^{u}_{e2k} - \Delta e^{0}_{e2k})$

$/(H/2),\ 1,\ 1\}$. Also, in a similar way, the load sub-vectors $\Delta \mathbf{f}_{1k}^{rf}$, $\Delta \mathbf{f}_{3k}^{rj}$, $\Delta \mathbf{f}_{1k}^{pj}$ and $\Delta \mathbf{f}_{3k}^{pj}$ can be written in the following form:

$$\Delta \mathbf{f}_{1k}^{rj} = EA_{\mathrm{r}} \mathbf{K}^{*} \mathbf{S}_{1rk}^{j-1} \qquad \Delta \mathbf{f}_{3k}^{rj} = EA_{\mathrm{r}} \mathbf{K}^{**} \mathbf{S}_{3rk}^{j-1} \qquad \Delta \mathbf{f}_{1k}^{pj} = EA_{\mathrm{p}} \mathbf{K}^{*} \mathbf{S}_{1pk}^{j-1} \qquad \Delta \mathbf{f}_{3k}^{pj} = EA_{\mathrm{p}} \mathbf{K}^{**} \mathbf{S}_{3pk}^{j-1}$$

where $\mathbf{S}_{1rk}^{j-1^T} = \left\{ \P e_{rk1}{}^{j-1},\ \P e_{rk2}{}^{j-1},\ 1 \right\}$, $\mathbf{S}_{3rk}^{j-1^T} = \left\{ \P e_{rk1}{}^{j-1},\ \P e_{rk2}{}^{j-1},\ 1,\ 1 \right\}$, $\mathbf{S}_{1pk}^{j-1^T} = \left\{ \P e_{pk1}{}^{j-1},\right.$ $\left.\P e_{pk2}{}^{j-1},\ 1 \right\}$, and $\mathbf{S}_{3pk}^{j-1^T} = \left\{ \P e_{pk1}{}^{j-1},\ \P e_{pk2}{}^{j-1},\ 1,\ 1 \right\}$. For a generic beam made up of several elements, using the usual technique of the finite element method for assembling and applying the boundary conditions, the system of linear algebraic equations can be obtained.

$$\underline{\mathbf{K}}_k^j \Delta \underline{U}_k^j = \Delta \underline{\mathbf{F}}_k^j \tag{2.23}$$

where $\underline{\mathbf{K}}_k^j$ represents the stiffness matrix, $\Delta \underline{U}_k^j =$ the unknown nodal displacement vector; $\Delta \underline{\mathbf{F}}_k^j$ is the load vector including the nodal force and prestressing force vectors of the beam in the time increment (k) for iteration (J). Obviously, the overall matrices $\underline{\mathbf{K}}_k^j$ and $\Delta \underline{\mathbf{F}}_k^j$ are assembled using the element's matrices in Eq.19. Once in any time increment the nodal displacement vector $\Delta \underline{U}_k^{j-1}$ is obtained using Eq.23 the overall stiffness matrix and load vectors will be modified and give a new nodal displacement vector $\Delta \underline{U}_k^j$. These procedures will be repeated until the difference between $\Delta \underline{U}_k^j$ and $\Delta \underline{U}_k^{j-1}$ is in neglected order. Then, one can get the real displacements, strains, and stress's distribution through the sections at time t_k.

3. CONCLUSIONS

A method for the time-dependent analysis of partially prestressed continuous composite beam with flexible shear connectors was presented taking into account the creep and shrinkage effects, the effect of progression of cracks and the stiffness of concrete between cracks. This method permits evaluating the time development of the displacements, strains, and stress's distribution due to the application of geometrical and static load for any load history.

4. REFERENCES

Bazant, Z. P. (1972). "Prediction of concrete creep effects using age adjusted effective modulus methods." *American Concrete Institution Journal*, 69(4), 212-217.

Dezi, L., and Tarantino, A. M. (1993). "Creep in composite continuous beams. I: Theoretical treatment." *Journal of the Structural Engineering*, Vol. 119, No. 7, 2095-2111.

Dezi, L., Leoni, G., and Tarantino, A. M. (1995). "Time dependent analysis of prestressed composite beams." *Journal of Structural Engineering*, Vol. 121, No. 4, 621-633.

Gilbert, R. I., and Bradford, A. (1995). "Time-dependent behaviour of continuous composite beams at service loads." *Journal of Structural Engineering*, Vol. 121, No. 2, 319-327.