SIZE EFFECT IN CONCRETE STRUCTURES: FAILURES, SAFETY AND DESIGN CODES

Z D E N Ě K P. B A Ž A N T

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SPONSORS: DoT – ITI, NSF

fib HUNGARIAN GROUP & TU BUDAPEST, MAY 10, 2016

Basic Facts about Quasibrittle Size Effect

FAILURE TYPES

a) Plastic

- **No localization, simultaneous, strengths are summed**
- **C.o.V. decreases with structure size as** $1/\sqrt{D}$
- **Gaussian distribution**

b) Brittle

- **Localizes, propagates, one element controls** $P_{\max}$
- **No decrease of C.o.V. with size** $D \Rightarrow$ **high scatter**
- **Weibull distribution**

c) Quasibrittle

Finite fracture process zone:

- charact. length ➡ Transitional SIZE EFFECT.

Small size: Quasi-Plastic. → Large size: Brittle.

QUASIBRITTLE MATERIALS

concrete *(archetypical)*

fiber composites

rocks

sea ice

toughened ceramics

rigid foams

wood

consolidated snow

particle board

paper, carton

cast iron

nanocomposites

metallic thin films

biological shells—nacre

mortar

masonry

fiber-reinforced concrete

stiff clays

silts, cemented sands

grouted soils

particle board

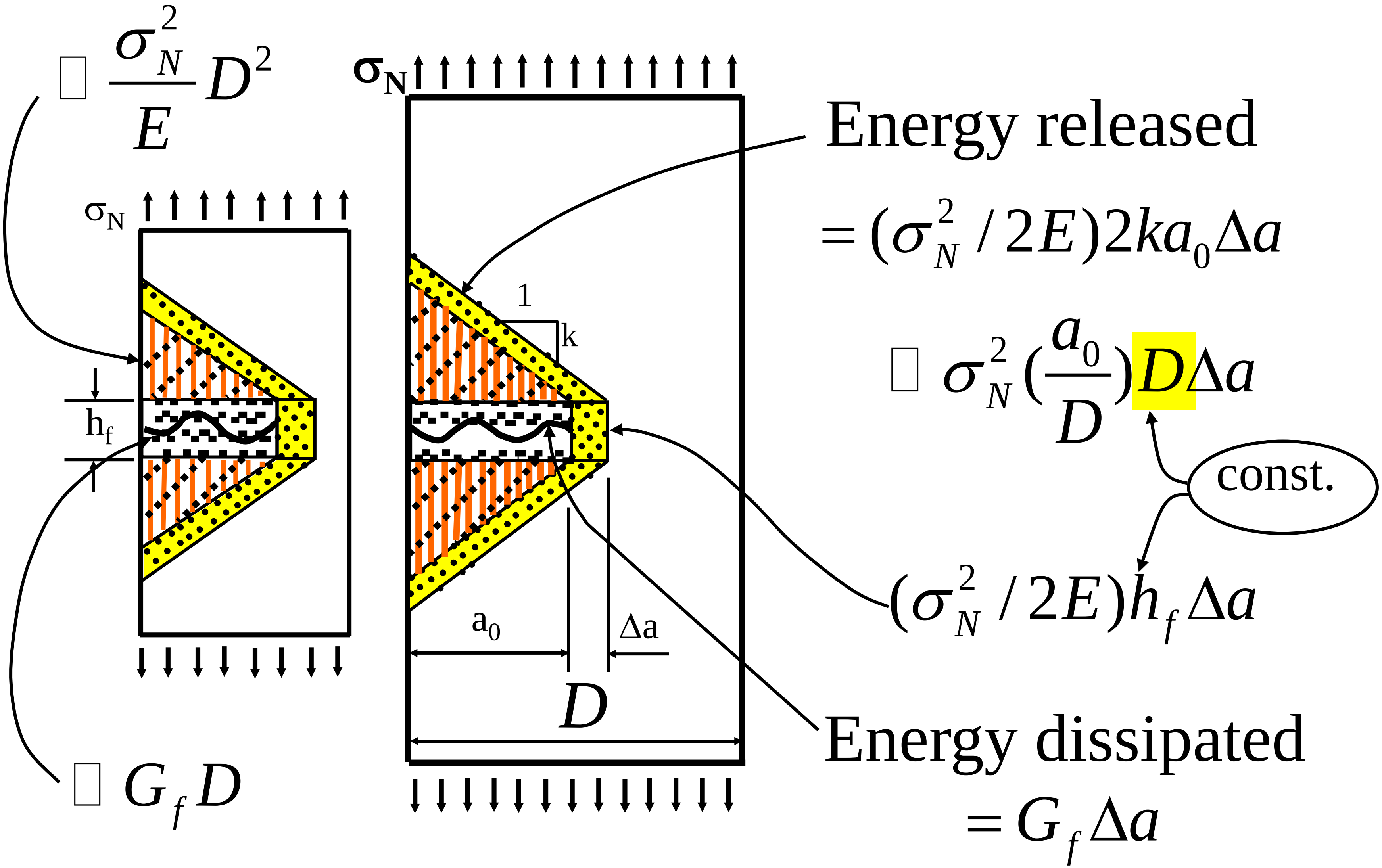
refractories

bone, cartilage

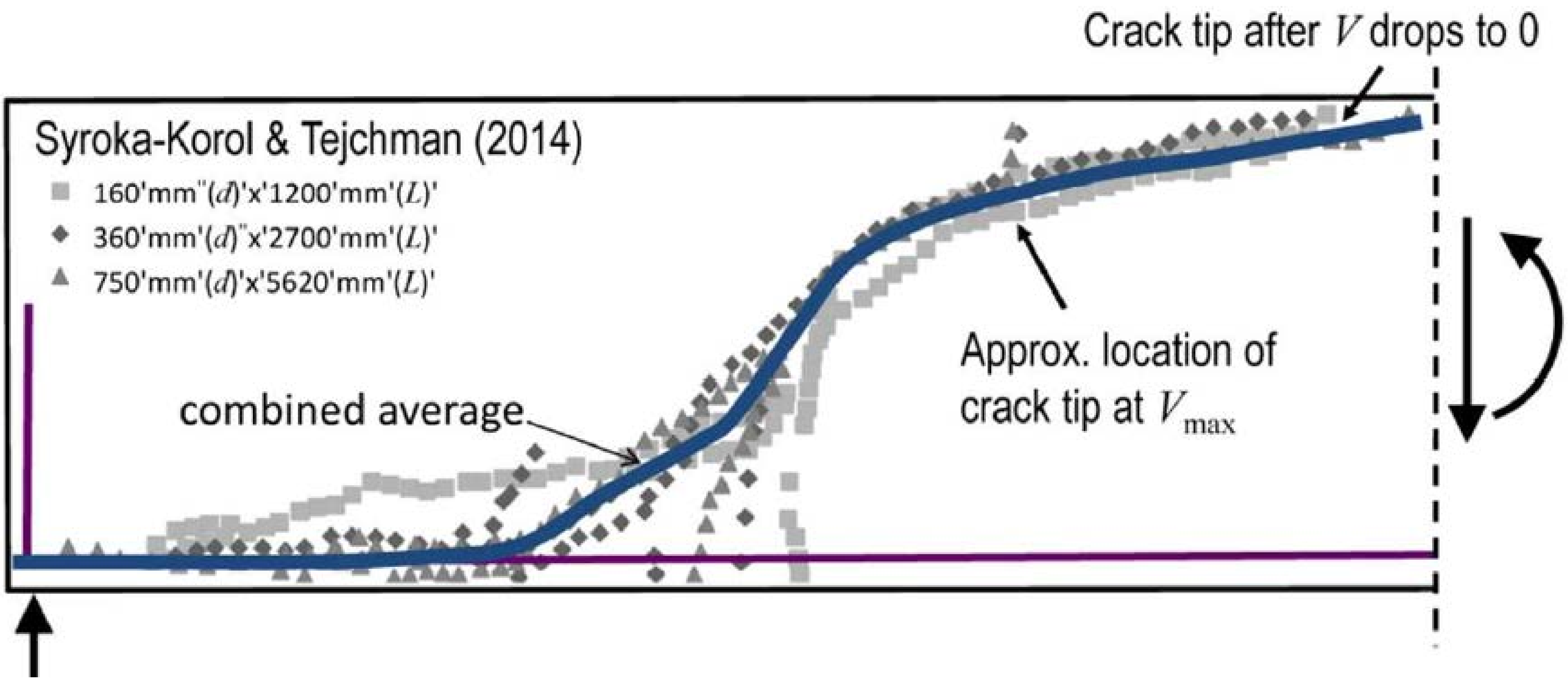
coal

modern tough alloys,…

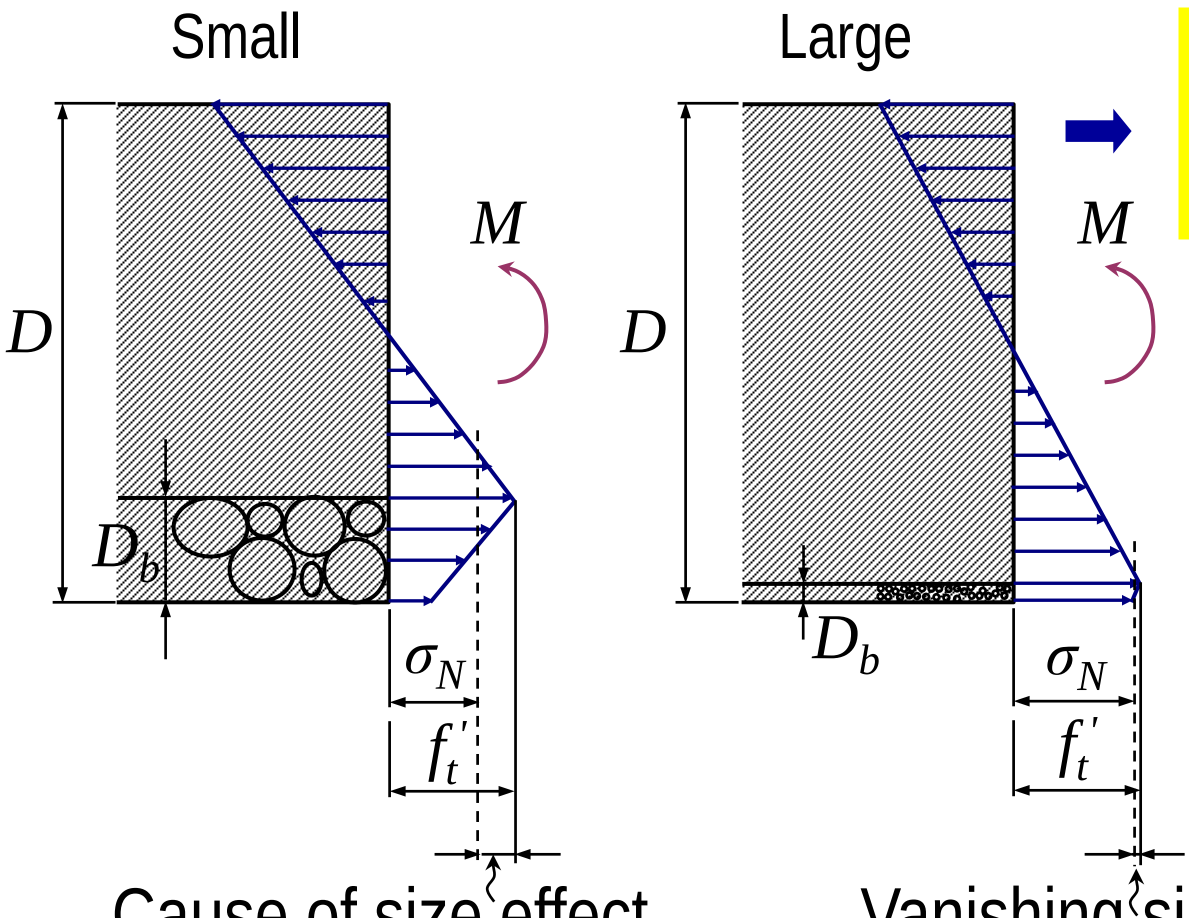
Intuitive Explanation of Energetic Size Effect (Type 2)



Geometric similarity of fractures at different size is a required hypothesis for Type 2 size effect. Here is a demonstration that it holds true for beam shear failures at different sizes.



Cause of Size Effect (of Type 1) for Failure at Crack Initiation



$$\sigma_N = \sigma_\infty \left(1 + \frac{rD_b}{D}\right)^{1/r}$$

(Alternative derivation: by fracture mechanics for a → 0)

Cause of size effect

(Can this be denied and replaced by fractals?)

Vanishing size effect, but <u>strength randomness</u> becomes important

Size Effect (of Type 2): Dimensional Analysis and Asymptotic Matching

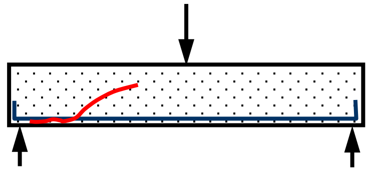
4 variables: f_t' [N/m^2], G_f [J/m^2], D [m], σ_N [N/m^2]

Characteristic material length implied (Irwin 1958): $l_0 = EG_f / f_t'^2$

Buckingham theorem:

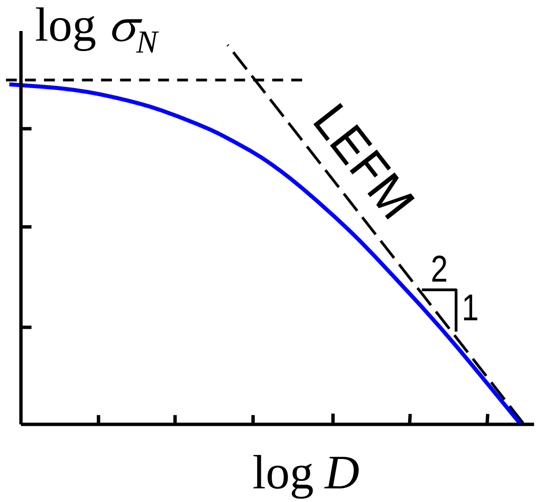
Max. load given by: $f(\Pi_1, \Pi_2) = 0$ or $f\left(\frac{\sigma_N^2 D}{f_t'^2 l_0}, \frac{\sigma_N^2}{f_t'^2}\right) = 0$

Large crack at max. load



If $D << l_0$: $f\left(\frac{\sigma_N^2}{f_t'^2}\right) = 0 \quad \rightarrow \quad \sigma_N \sim f_t' = \text{const.}$

If $D >> l_0$: $f\left(\frac{\sigma_N^2 D}{f_t'^2 l_0}\right) = 0 \quad \rightarrow \quad \sigma_N \sim \sqrt{EG_f / D} \sim D^{-1/2}$



Expansion of $f(\Pi_1, \Pi_2) = 0$ up to 2nd order terms yields approx. size effect law:

$$\sigma_N = Bf_t'(1 + D / D_0)^{-1/2}$$

Bazant, PNAS 2004

Derivations of Size Effect Law

1. **Analytical:**

- Simplified energy release analysis
- Simplified contour integration of J-integral
- Dimensional analysis with asymptotic matching
- Asymptotic transformations of diff.eqs. with boundary conditions
- Asymptotic analysis of equivalent LEFM
- Asymptotics of cohesive crack model (smeared-tip method)
- Asymptotic expansion of J-integral
- Deterministic limit of probabilistic nonlocal theory

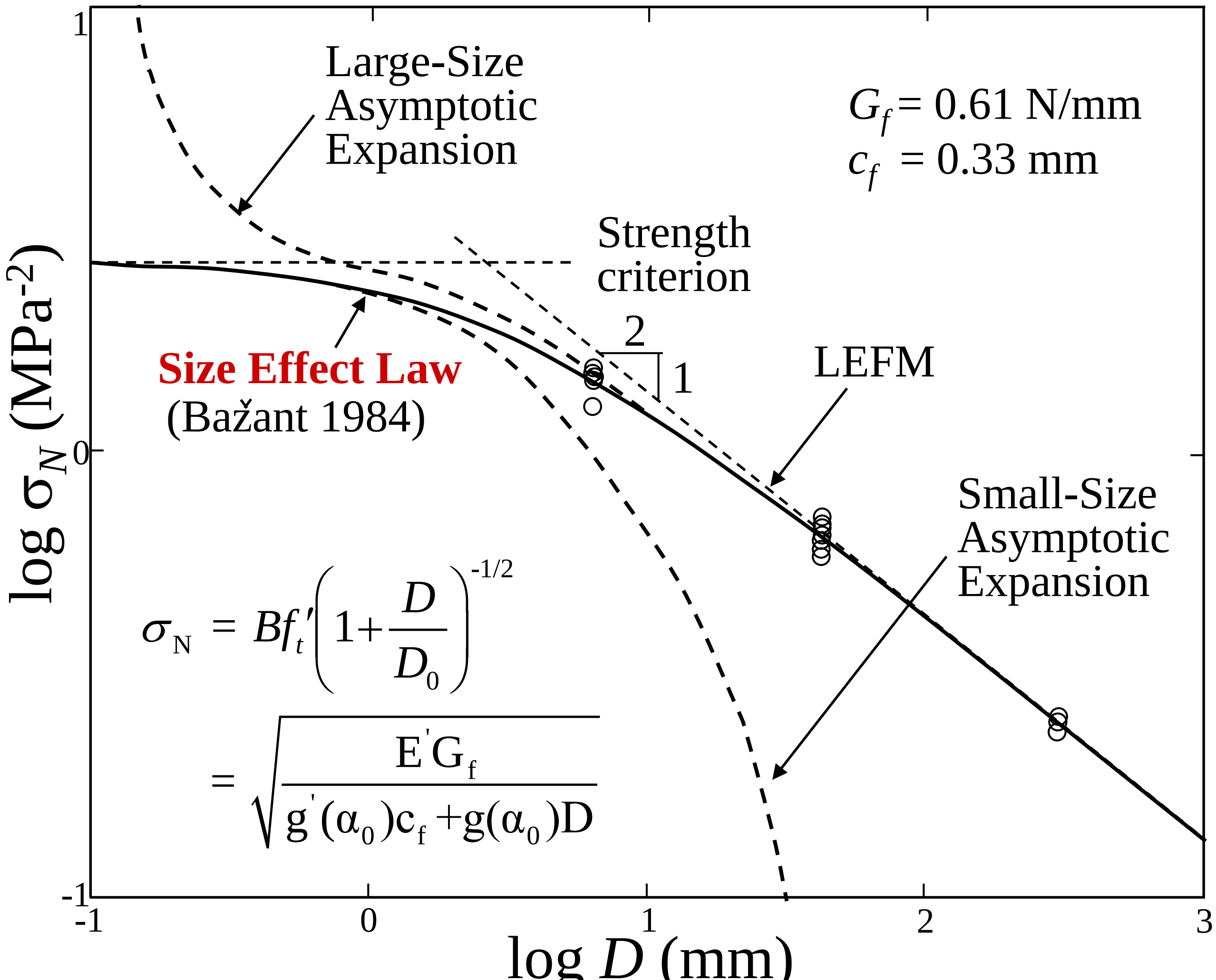
2. **Numerical:**

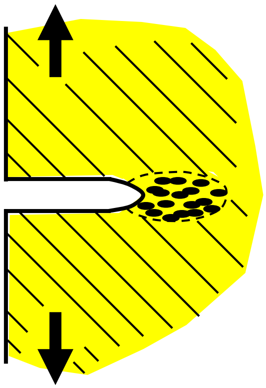
- FEM with strongly nonlocal damage models
- FEM with gradient (weakly nonlocal) models
- Random Lattice Discrete Particle Model
- Limit of nonlocal probabilistic FEM

MATERIALS:
Concretes, rocks, sea ice, toughened ceramics, fiber composites, brittle foams, wood, snow (avalanches), particle board, paper, …

Asymptotic Matching

Size Effect of Divinycell H100 Foam

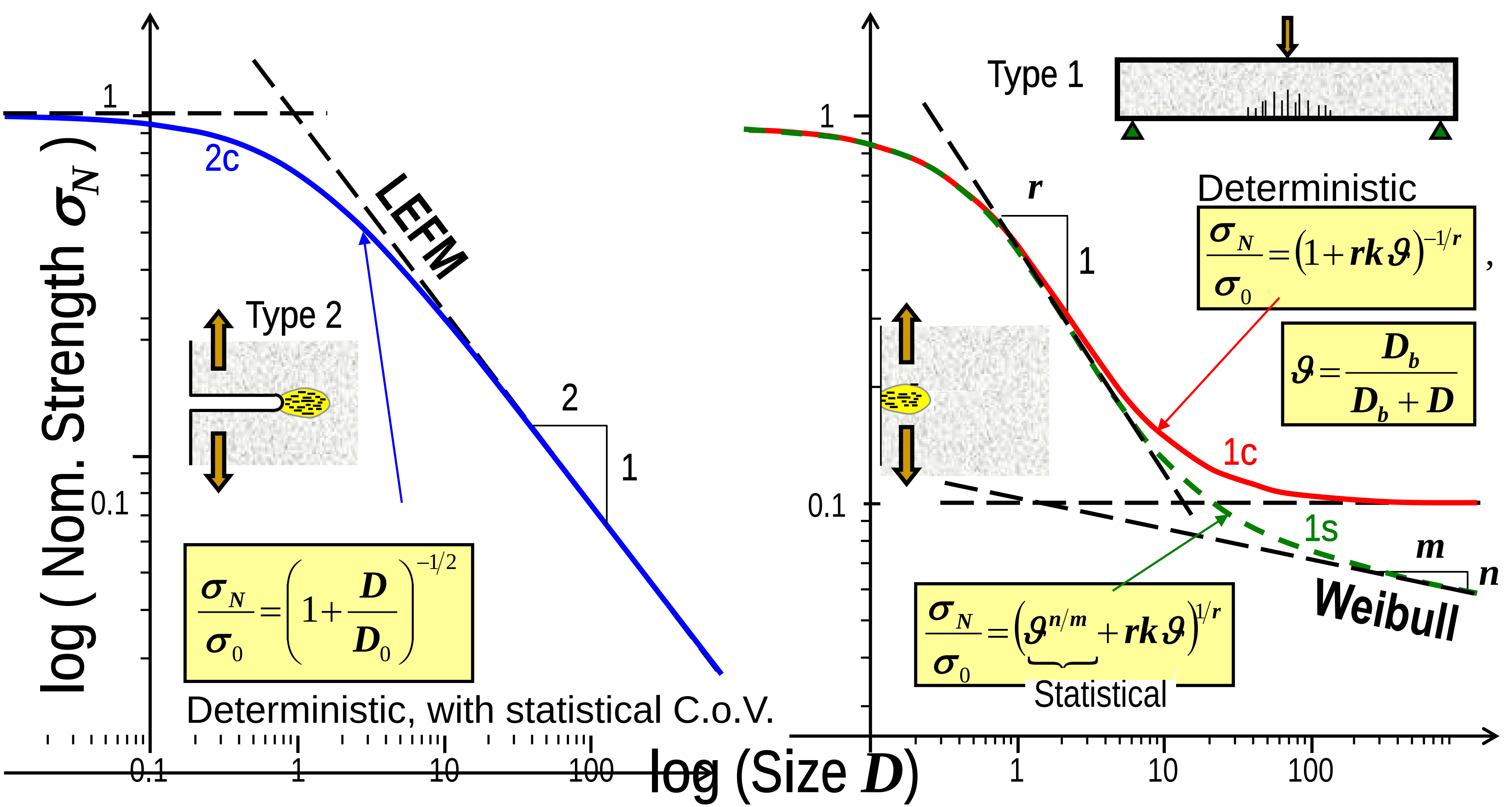




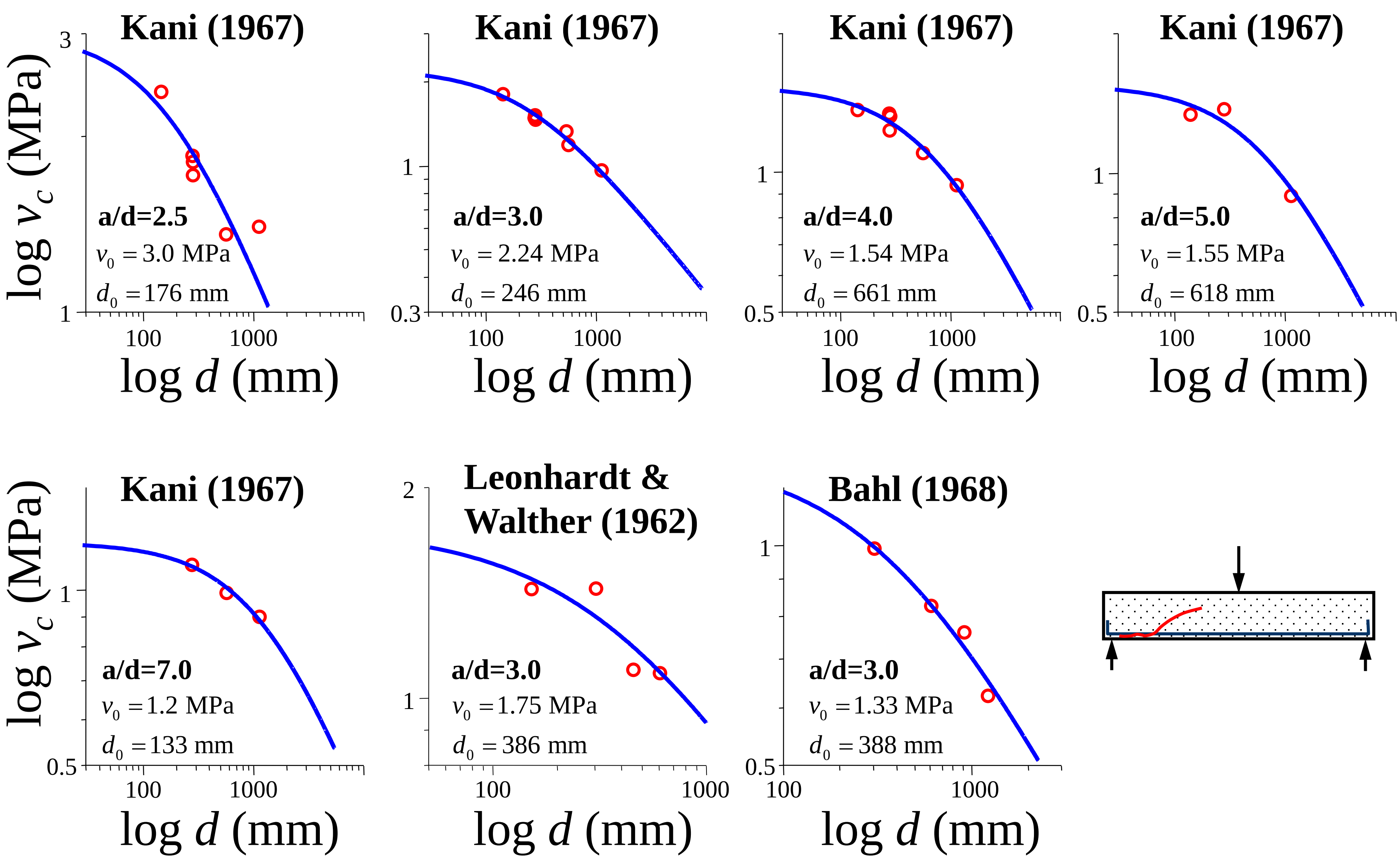
Easy **Hard !** **Easy**

Energetic (Quasibrittle) Mean Size Effect Laws and Statistical Generalization

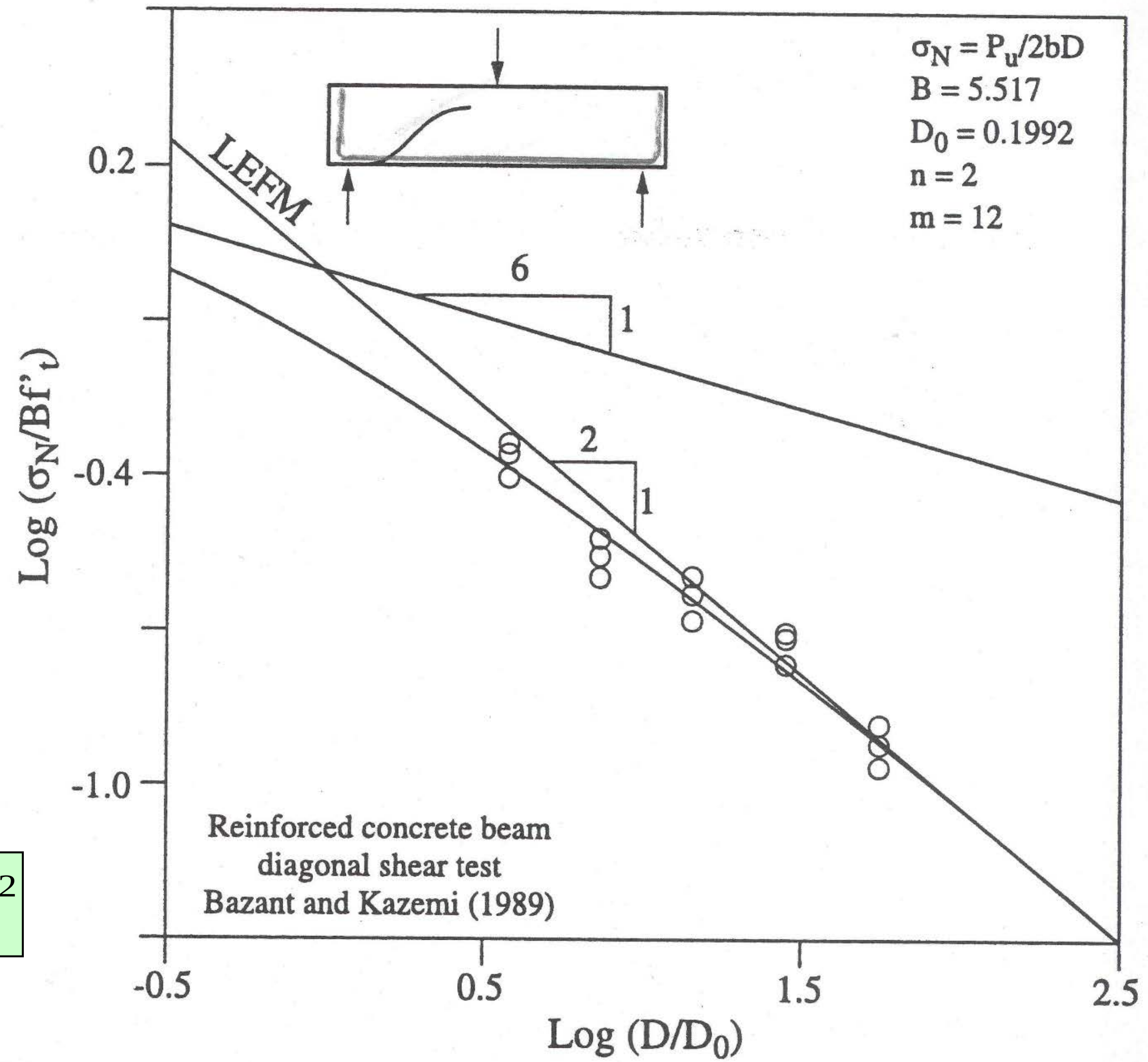
2c – based on cohesive crack model, **1s – statistical generalization**



Classical Narrow-Range Test Data for Size Effect on Shear Strength of R.C. Beams without Stirrups



Reduced-Scale Tests of Beam Shear Failure at Northwestern (aggregate < 48 mm)



$$\sigma_N = \sigma_p (1 + d / d_0)^{-1/2}$$

Incorporating the size effect into design code formulas for reinforced concrete is easy:

Multiply the formula for the strength contribution of concrete by the **size effect factor**:

$$\vartheta = \frac{1}{\sqrt{1 + d / d_0}}$$

(type 2 size effect)

Design Formula for Shear in R.C. Beams

- Proposed in *fib* 2010 Draft

$$V_{Rd,c} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c}$$

$$k_v \begin{cases} = \dfrac{200}{1000 + 1.3z} \leq 0.15 & \text{if } \rho_w = 0 \\ = 0.15 & \text{if } \rho_w \geq 0.08\sqrt{f_{ck}} / f_{yk} \end{cases}$$

- Proposed by ACI-446 (fracture mechanics based)

$$v_c = \mu \rho_w^{3/8} \left(1 + \frac{d^{0.4}}{a^{0.4}}\right) \sqrt{\frac{f_c'}{1 + d/d_0}}, \quad d_0 = \kappa \ f_c'^{-2/3}$$

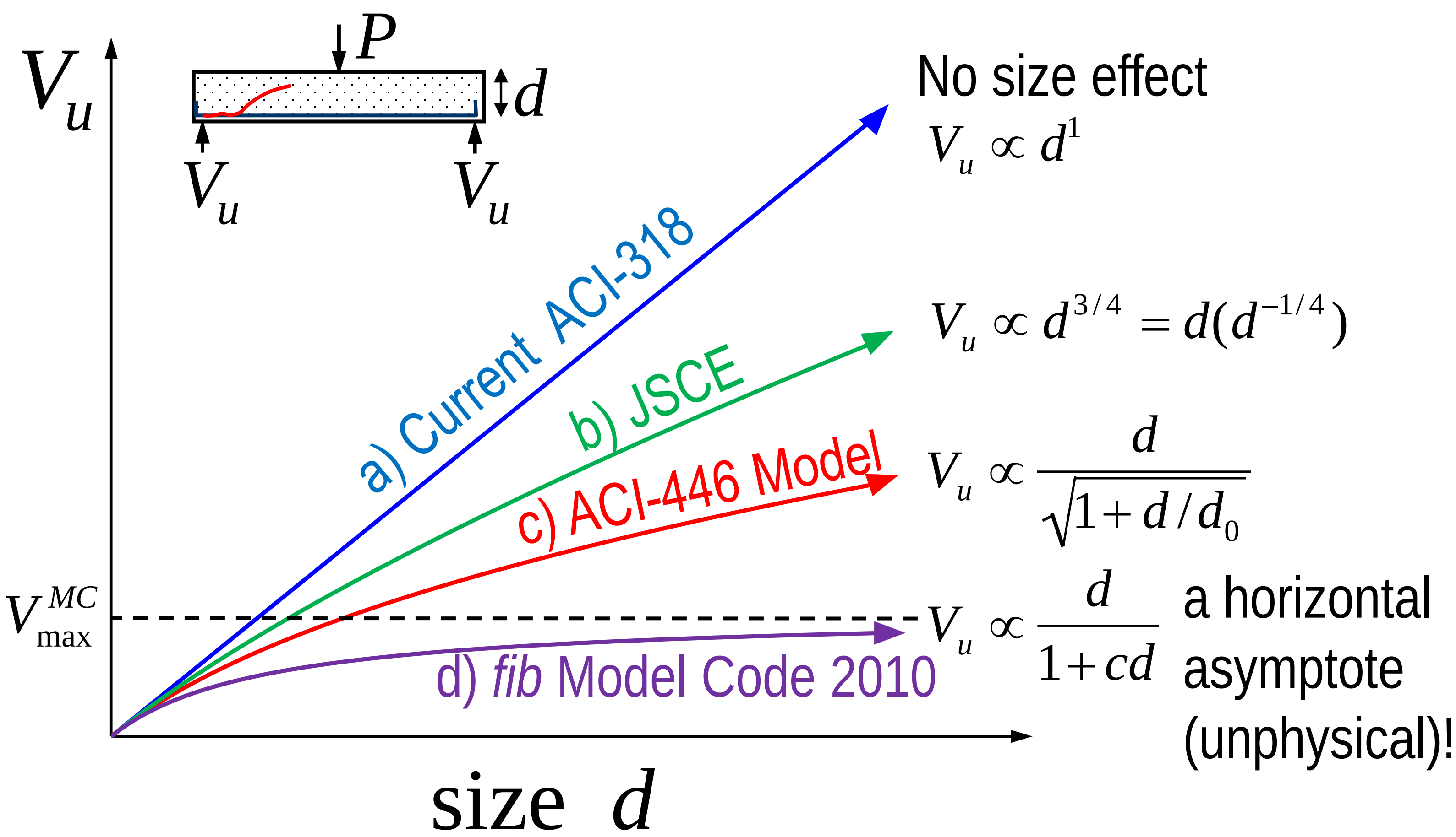
$$\text{where } \kappa = 3800\sqrt{d_a} \text{ if } d_a \text{ is known}, \quad \kappa = 3330 \text{ if not}$$

DEFICIENCIES:

1) Invalid derivation from *Modified Compression Field Theory, MCFT,* based on plasticity for crack initiation.
2) Effects of ρ_w and a/d ignored.
3) No size effect if minimum shear reinforcement exists
4) etc.

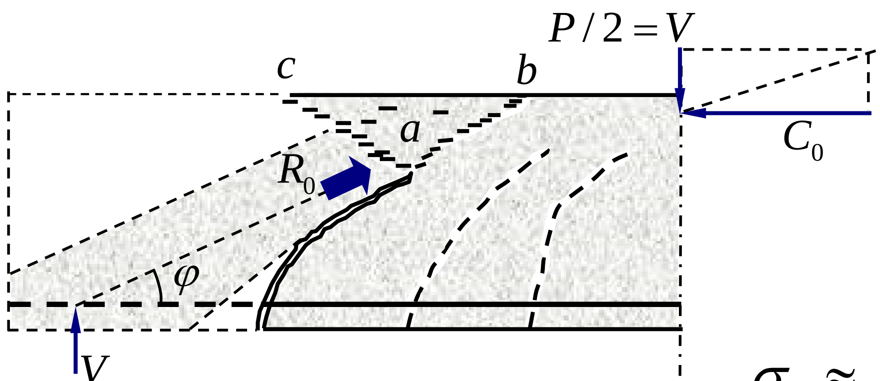
If stirrups ≥ minimum stirrups, the size effect still exists, though pushed into larger sizes (d_0 will increase by about one order of magnitude).

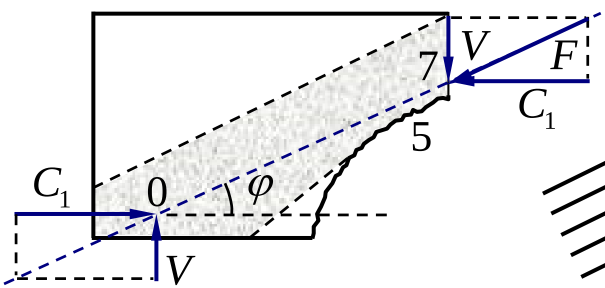
Comparison of
CURVES OF SHEAR FORCE $V_u = b_w\, d\, v_u$
vs. SIZE d in current codes

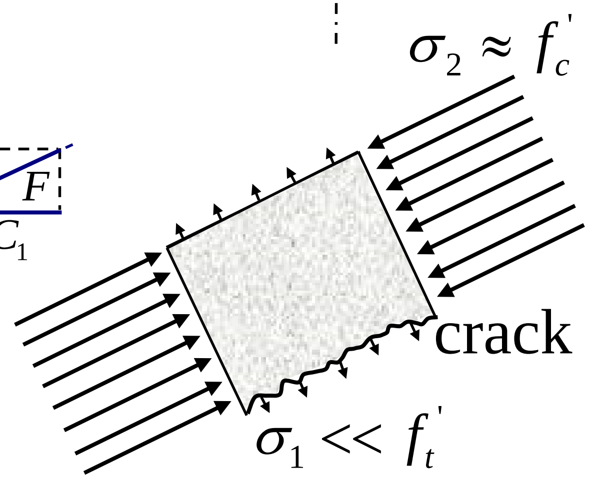


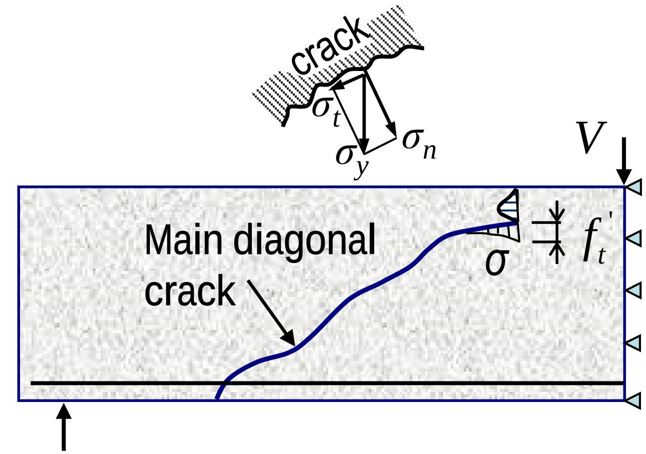
Note: The curves are scaled to the same initial tangent

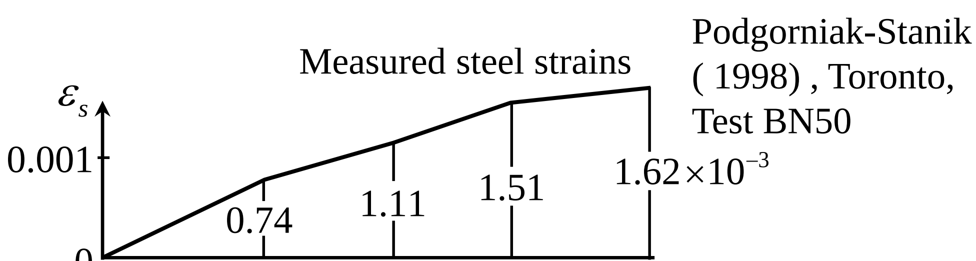
Stress transmitted across crack is not the reason







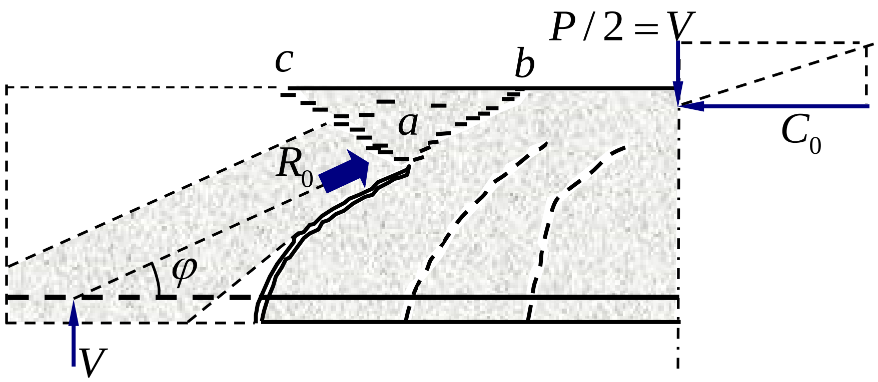




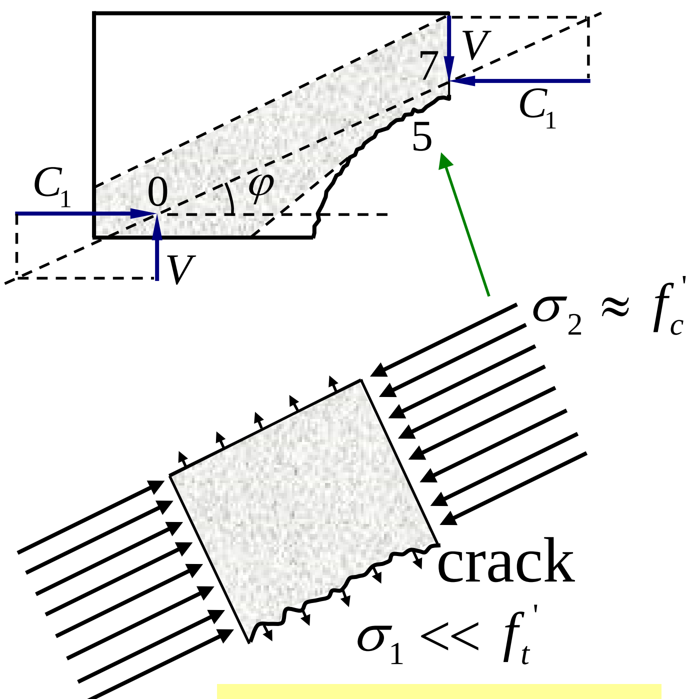
For small beams, the contribution of **crack-bridging stress** is significant, i.e., 40%; while **for 1.8 m deep beams**, it is negligible, i.e., 9%.

Failure Mechanism in Shear Test of Large Beam

Toronto, 1.89 m deep

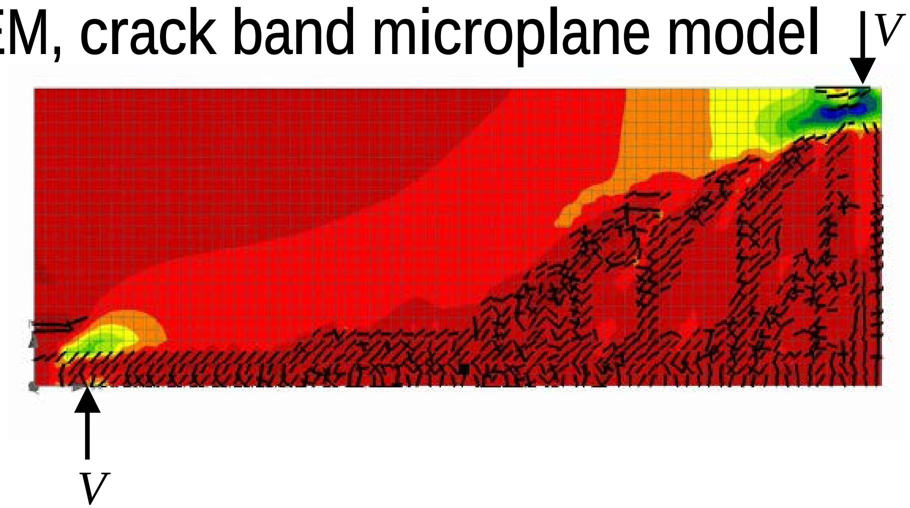


Compression resultant location

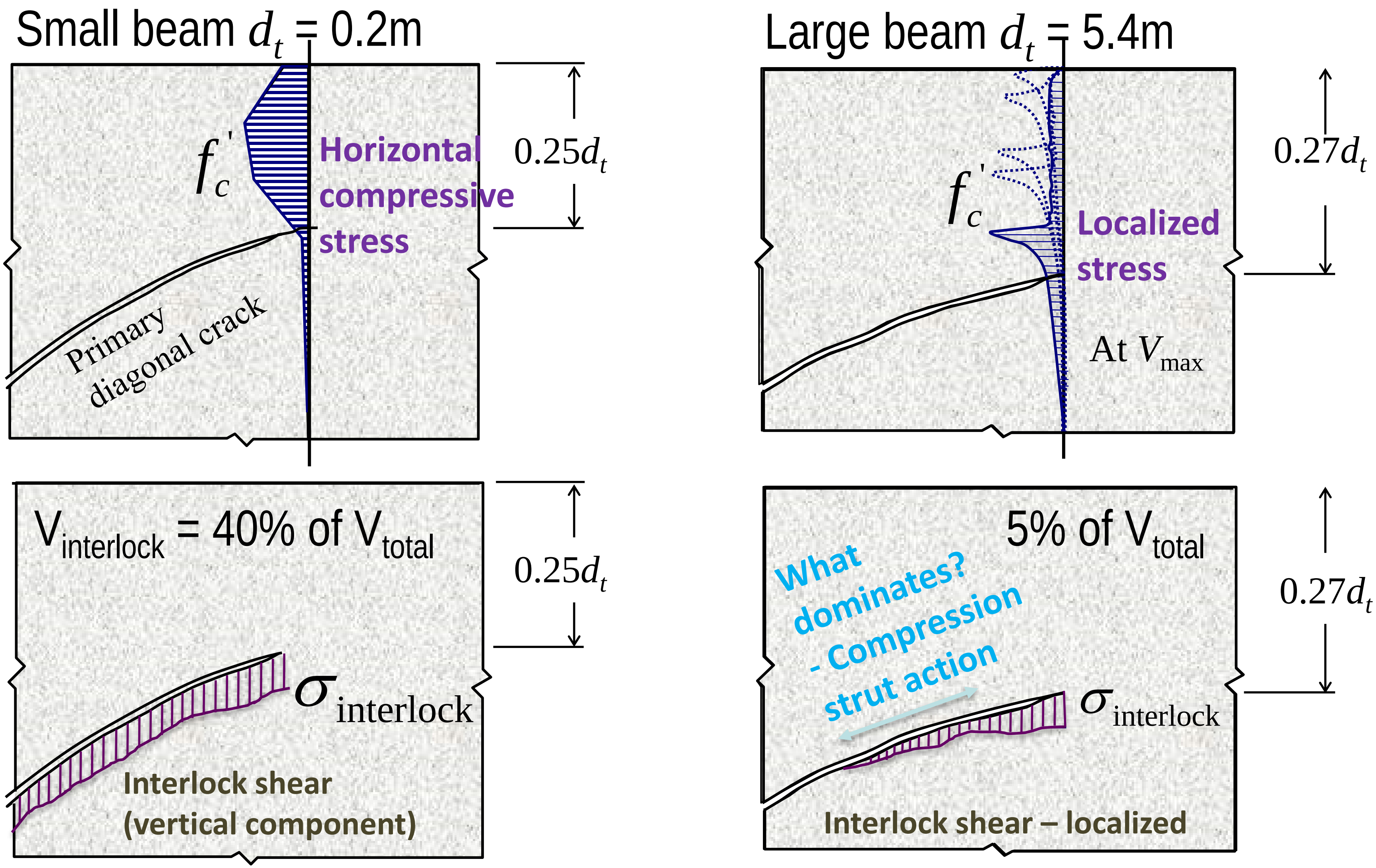


Negligible shear stresses on crack face !

FEM, crack band microplane model



EXPLANATION: In larger beams, f_c' doesn't get mobilized through the whole cross section

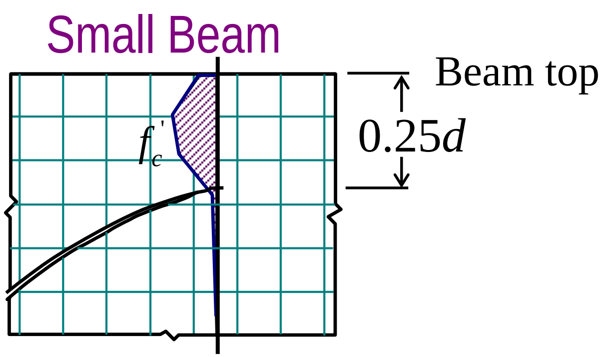


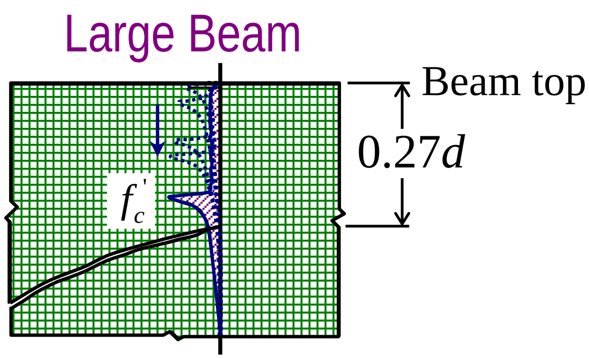
Stresses calculated by microplane crack band model calibrated by Toronto tests

Problems with Explaining the Size Effect by Reduction in Interface Shear Transfer Resistance (Collins et al.)

- Conflicts with **dimensional analysis** based on known **asymptotic properties** (the asymptotic slope of -1 is excessive, thermodynamically impossible).
- Is **not general**: Doesn't work for **other failure types and materials** with the same kind of size effect (e.g. punching shear or compressive crushing).
- In large beams, the tensile **cohesive stresses along the diagonal crack** at peak load are **negligible** compared to the compressive stress parallel to the crack.
- In large beams, the **interface shear due to aggregate interlock** contributes only a **minor part** to the total shear strength, although it has an significant effect in small beams.
- **Localization**, with increasing size, of the **compressive stress profile** across the ligament above the tip of the diagonal crack leads to **compressive crushing of concrete** at peak load.
- **Crack spacing** is a **secondary influence**, not the primary cause of size effect.

Compressive stress distribution

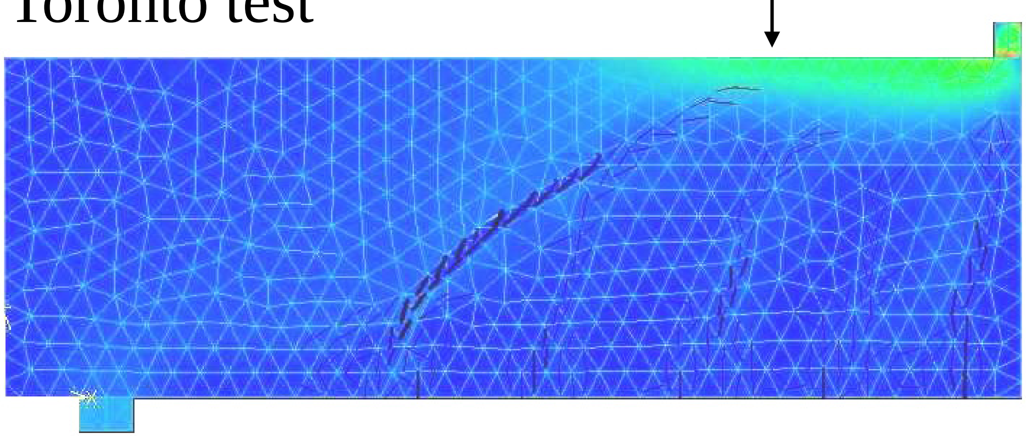


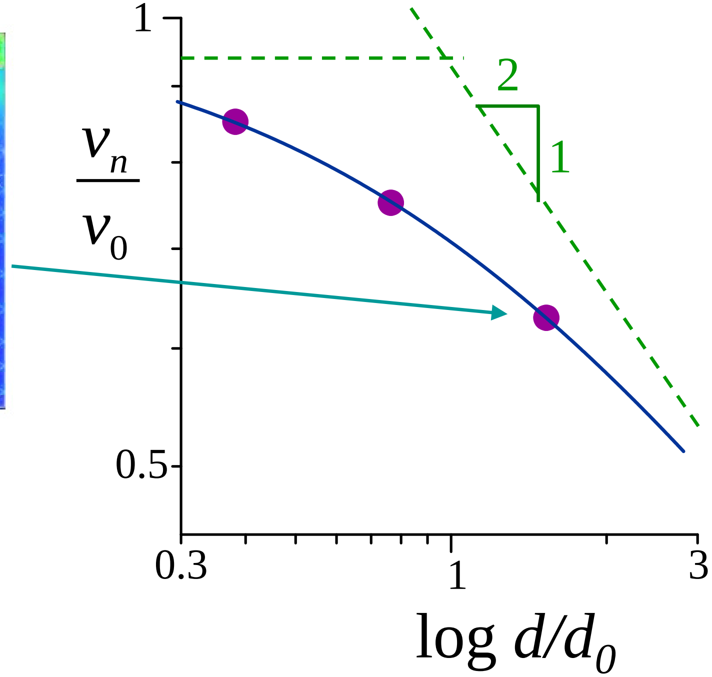


Size Effect in Beam Shear by Crack-Band Microplane Finite Element Model

Simulation of Toronto test

Compression splitting & crushing





Fracture mechanics based finite element simulations support the size effect law:

$$v_n = \frac{v_0}{\sqrt{1 + d/d_0}}$$

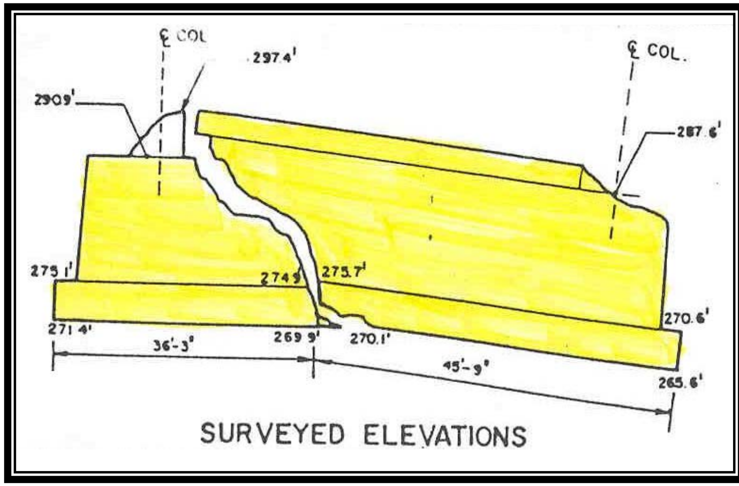
$$v_0 = 1.80 \text{ MPa}$$

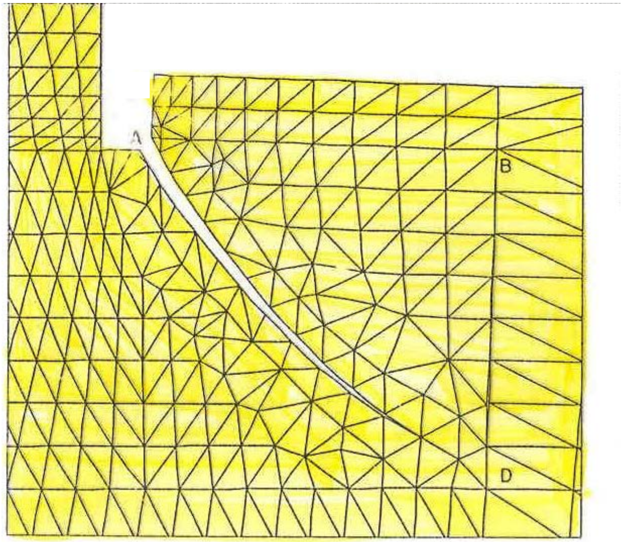
$$d_0 = 2.47 \text{ m}$$

Structural Failures with Evidence of Size Effect

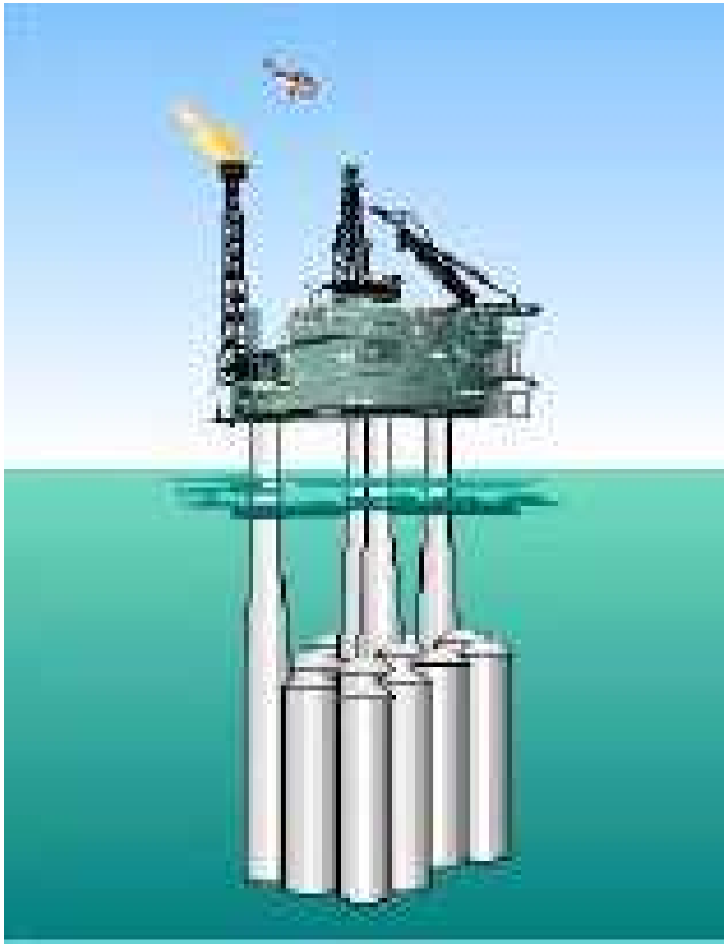
Schoharie Creek Bridge

N.Y. Thruway, 1987





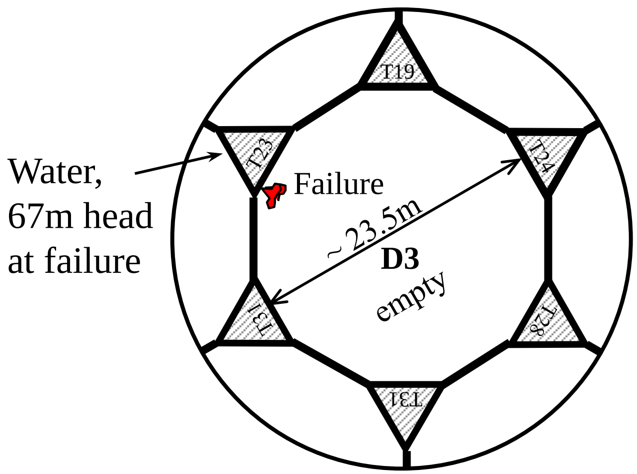
Sleipner A Platform, Norway



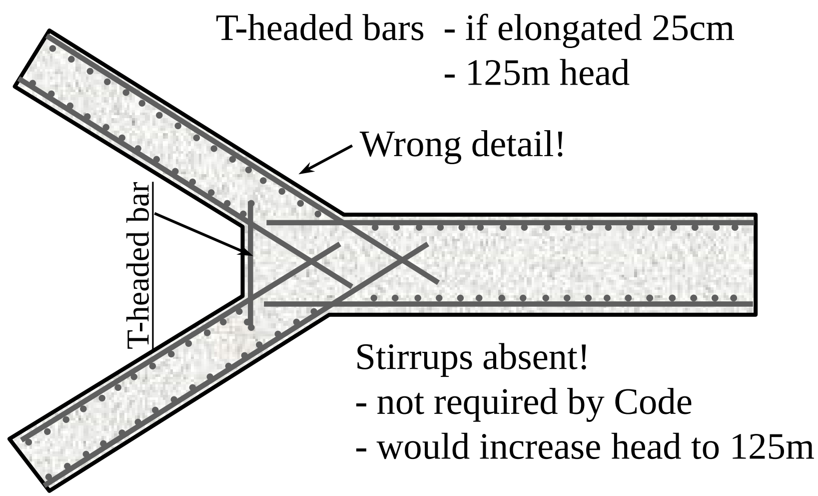
Sank 23 Aug, 1991

~ 82 m deep water
~ 190 m tall

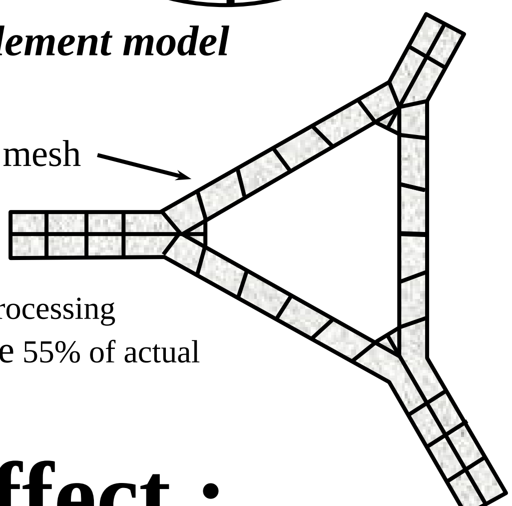
Failure of Sleipner A Platform



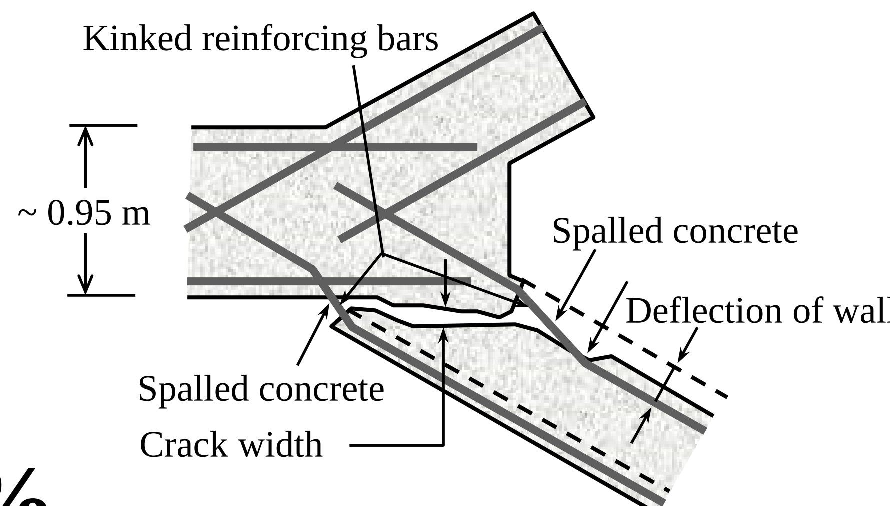
Finite element model



Assumed failure mode



With post-processing
- shear force 55% of actual



Posterior p_{max} ~ 0.8 - 1.2 $p_{applied}$!

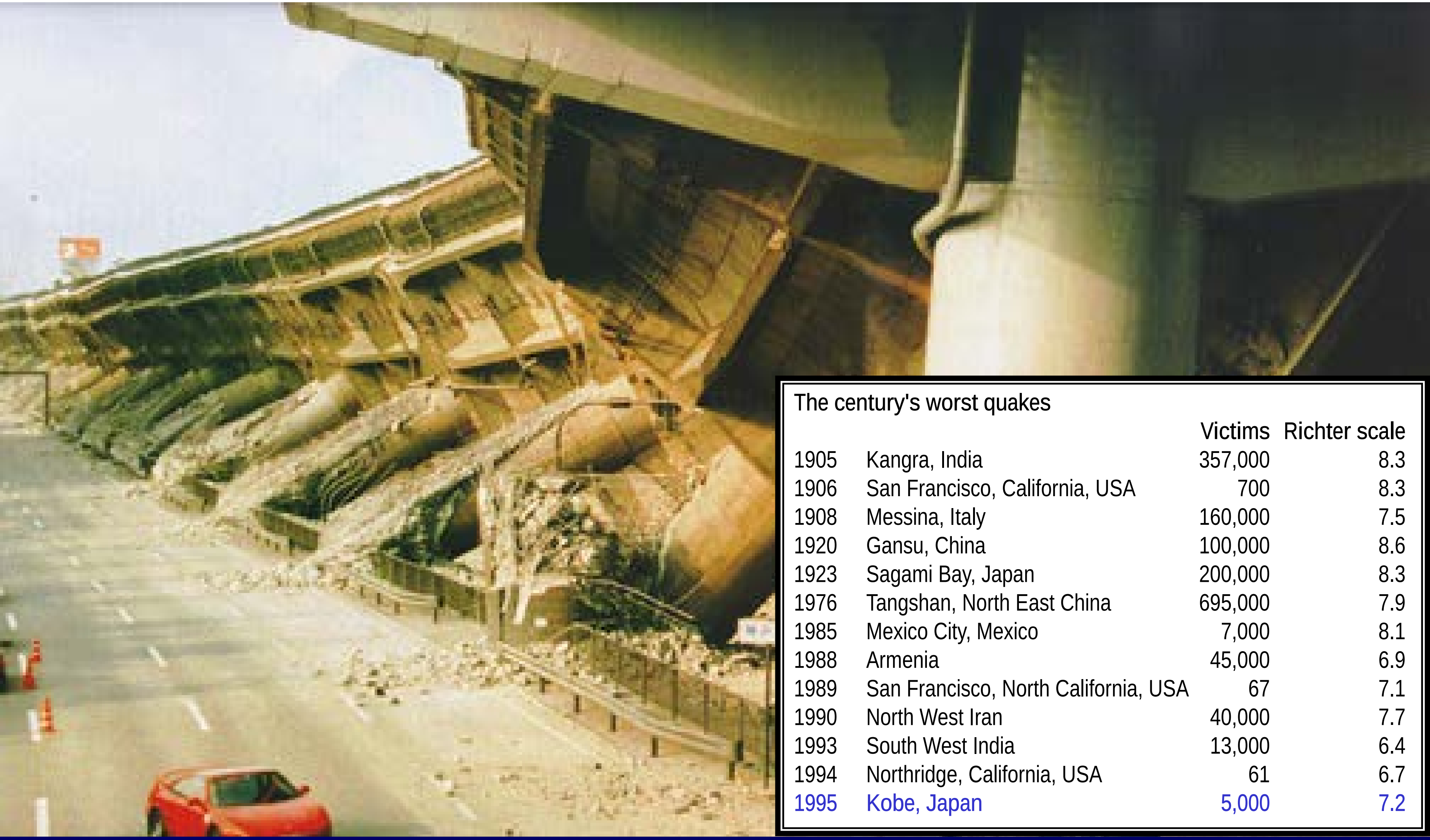
Size Effect :

f_t' reduced by 40%

Kobe (Hyogo-Ken Nambu) Earthquake, 1995, Hanshin Viaduct

- size effect due to compression fracture

Kobe Earthquake 1995, Hanshin Viaduct

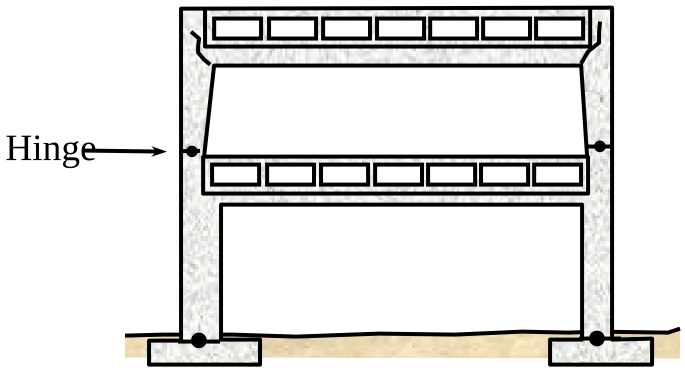


The century's worst quakes			
		Victims	Richter scale
1905	Kangra, India	357,000	8.3
1906	San Francisco, California, USA	700	8.3
1908	Messina, Italy	160,000	7.5
1920	Gansu, China	100,000	8.6
1923	Sagami Bay, Japan	200,000	8.3
1976	Tangshan, North East China	695,000	7.9
1985	Mexico City, Mexico	7,000	8.1
1988	Armenia	45,000	6.9
1989	San Francisco, North California, USA	67	7.1
1990	North West Iran	40,000	7.7
1993	South West India	13,000	6.4
1994	Northridge, California, USA	61	6.7
1995	Kobe, Japan	5,000	7.2

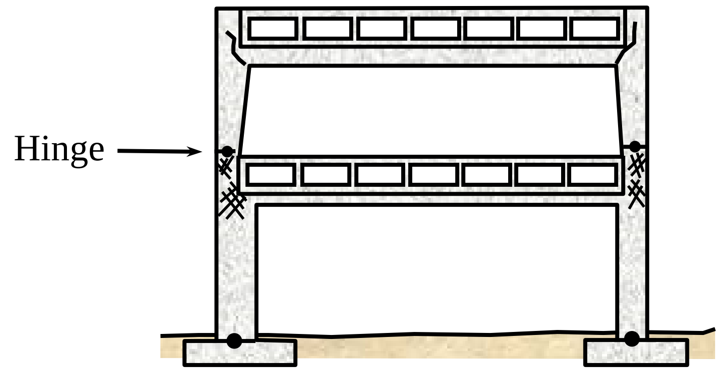
- Size effect due to compression fracture (in bending)

Cypress Viaduct (1989)

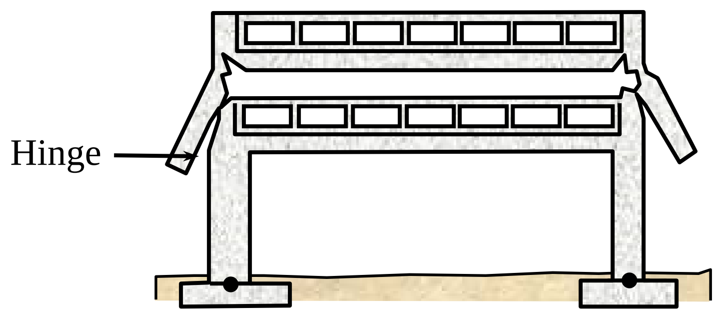
Nimitz Freeway, Oakland, CA



Cypress Structure :
Typical Bent with Hinged Frames



Crack Initiation at Hinge Location



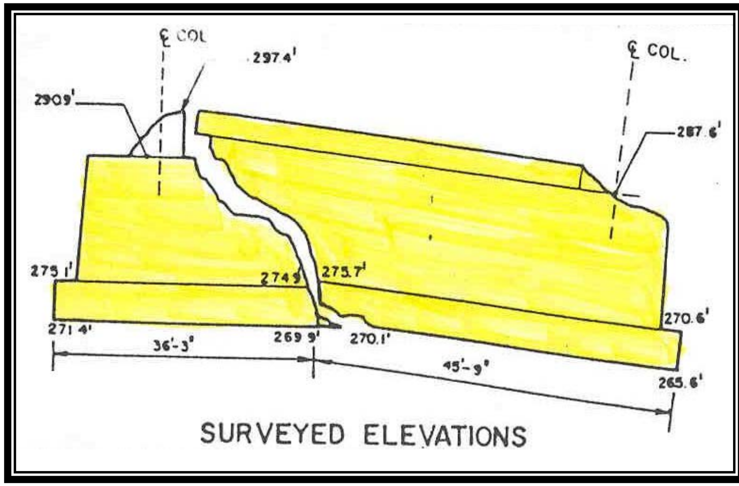
Typical Failure Mode of Bent

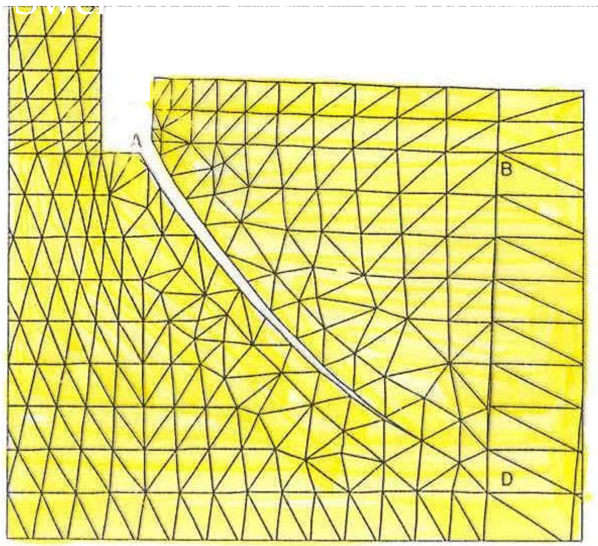
After *Salvadori*

Loma Prieta Earthquake

Schoharie Creek Bridge

N.Y. Thruway, 1987





Shear Failure—Size effect was a major factor

Blvd. de la Concorde, Laval, Quebec, North suburb of Montreal, Sept. 30, 2006

Wilkins Air Force Depot Warehouse, Shelby, Ohio

Failed 1955

Beam Depth: 0.914 m

Koror-Babeldaob Bridge in Palau

AA Yee, ACI Concr.Int. ,June1979,22-23

Built 1977, failed 1996. Max. girder depth 14 m, span 241 m (world record).

Koror-Babeldaob Bridge in Palau

Built 1977, failed 1996.

Our Verdict: Compression and shear failure, with wave from prestress failure, after creep & vertical prestress loss.

Size effect must have been strong.

Reappraisal of Some Structural Catastrophes:

SIZE EFFECT WAS AN IMPORTANT FACTOR!

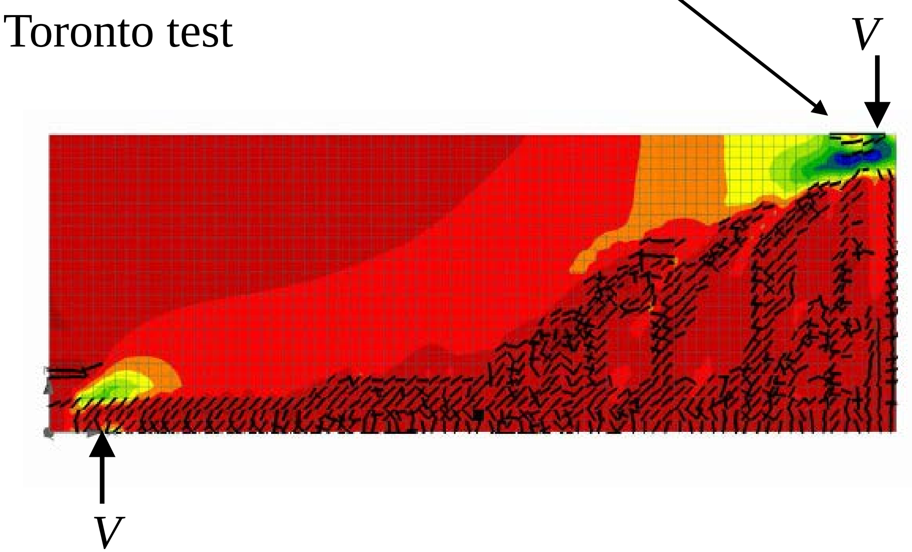
	Strength Reduction Due to Size Effect
Plain Concrete :	
• Malpasset Dam, France (failed 1959)	77%
• St. Francis Dam, L.A. (failed 1928)	60%
• plinth of Schoharie Creek Bridge, 1987	46%
Reinforced concrete :	
• Cypress Viaduct column, Oakland, 1989 earthquake	30%
• Hanshin Viaduct columns, Kobe, 1995 earthquake	38%
• bridge columns, L.A., 1994 Norridge earthquake	30%
• Sleipner A Oil Platform, plate shear, Norway, 1991	34%
• Warehouse, beam shear, Wilkins AF, Shelby, OH ‘55	32%
• record-span box girder, Palau , failed 1996 - prelim.:	?>50%
• Laval Overpass, Montreal, beam shear, 2006	>40%

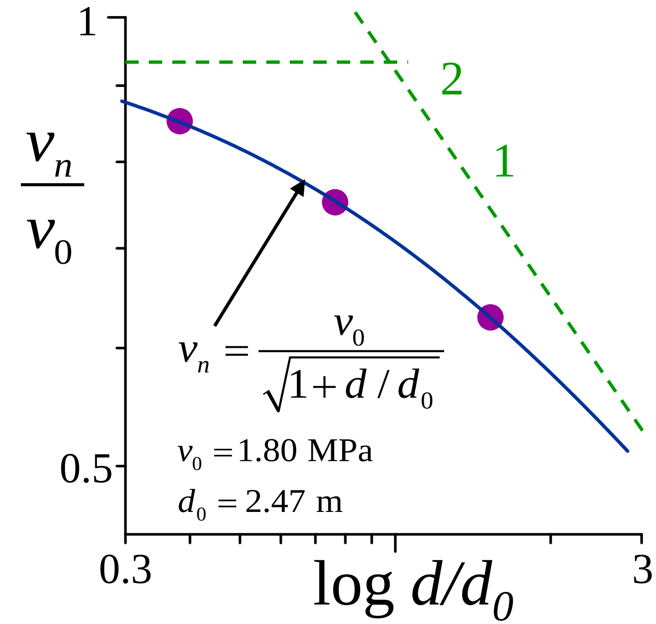
Evidence from Individual Laboratory Tests on the Same Concrete

Stress Intensity Field

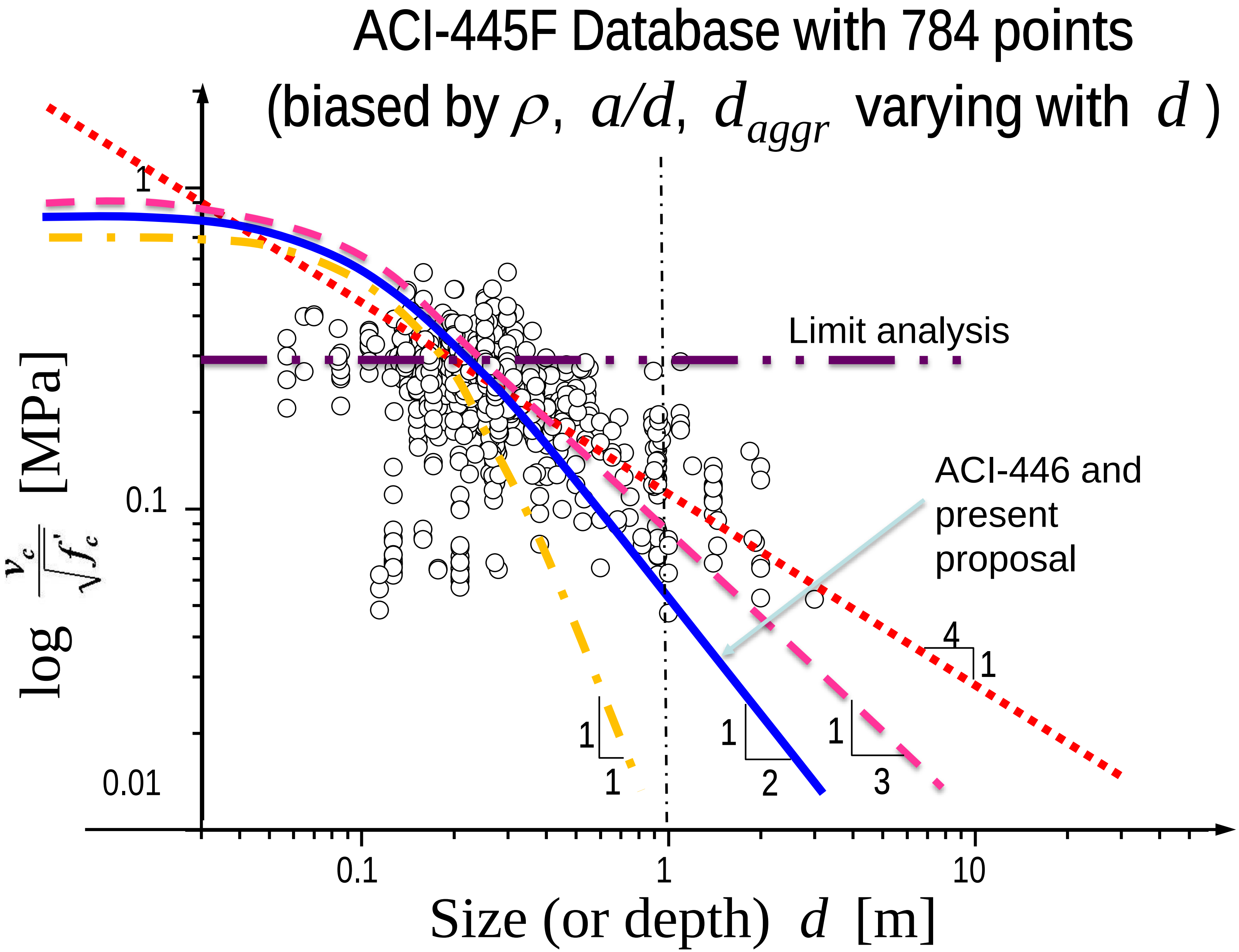
Compression
splitting & crushing

Toronto test



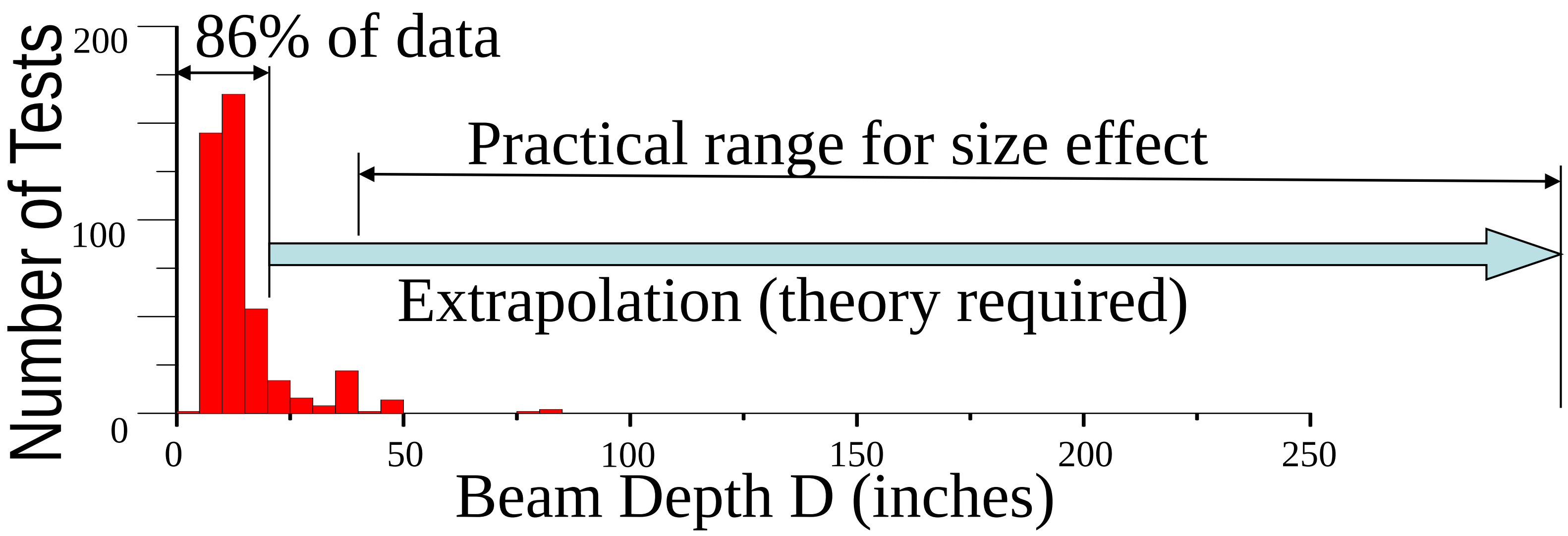


Evidence from Large Worldwide Laboratory Database

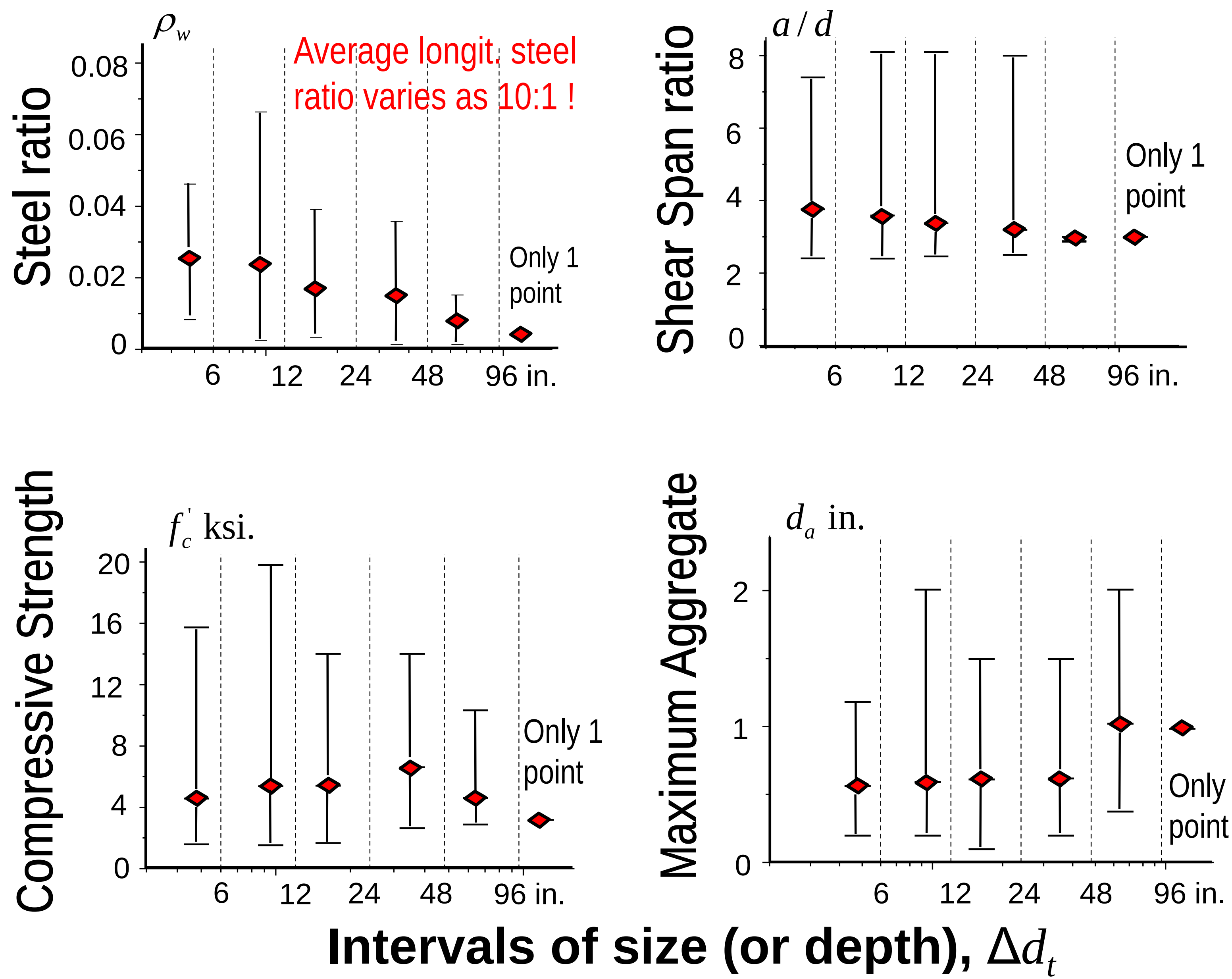


ACI-445F Database with 784 points
(biased by ρ, a/d, d_aggr varying with d)
Limit analysis
ACI-446 and present proposal
log $\frac{v_c}{\sqrt{f'_c}}$ [MPa]
Size (or depth) d [m]
1
0.1
0.01
0.1
1
10
4
1
1
1
1
2
1
3

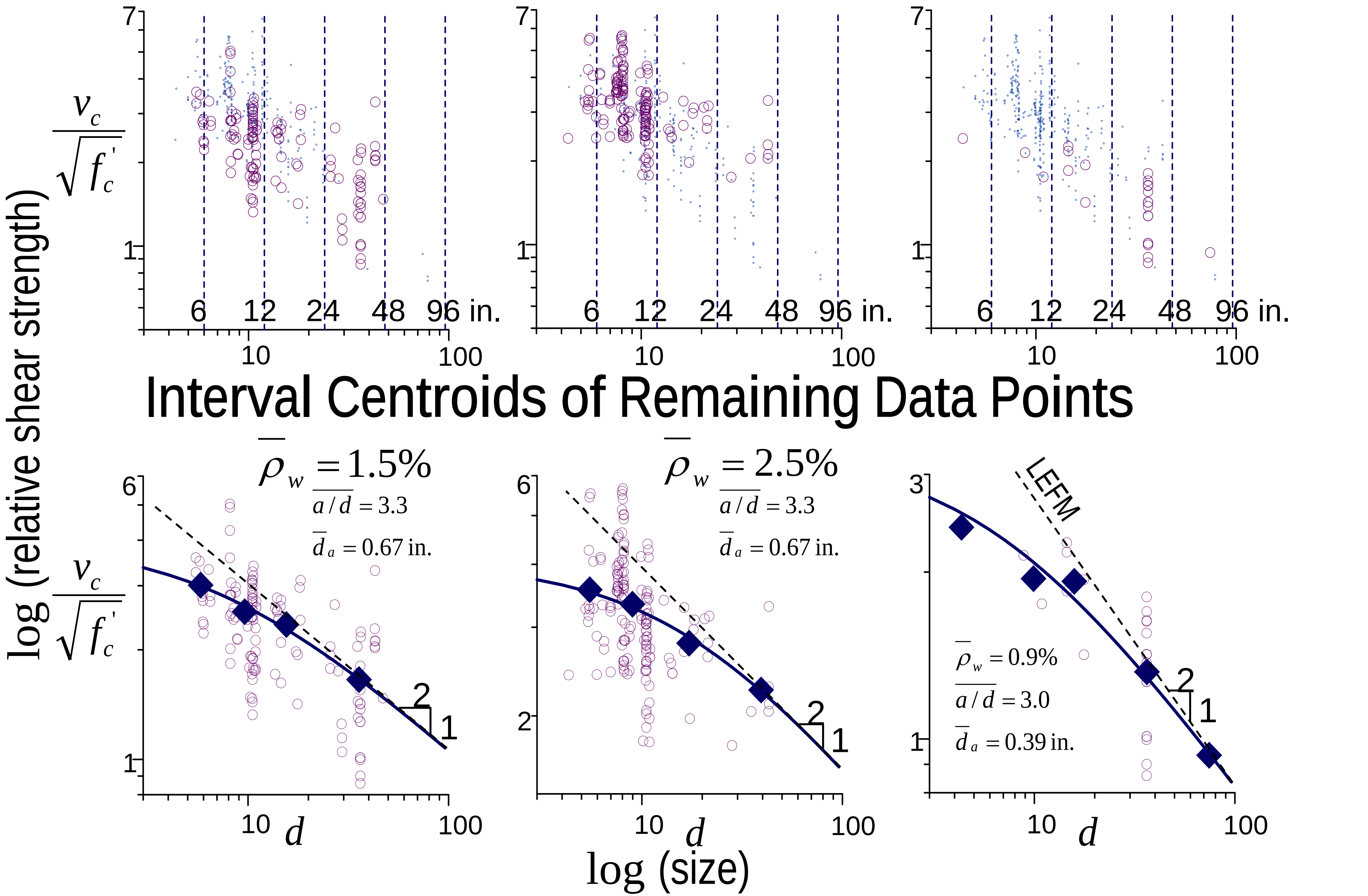
Beam Shear Histogram



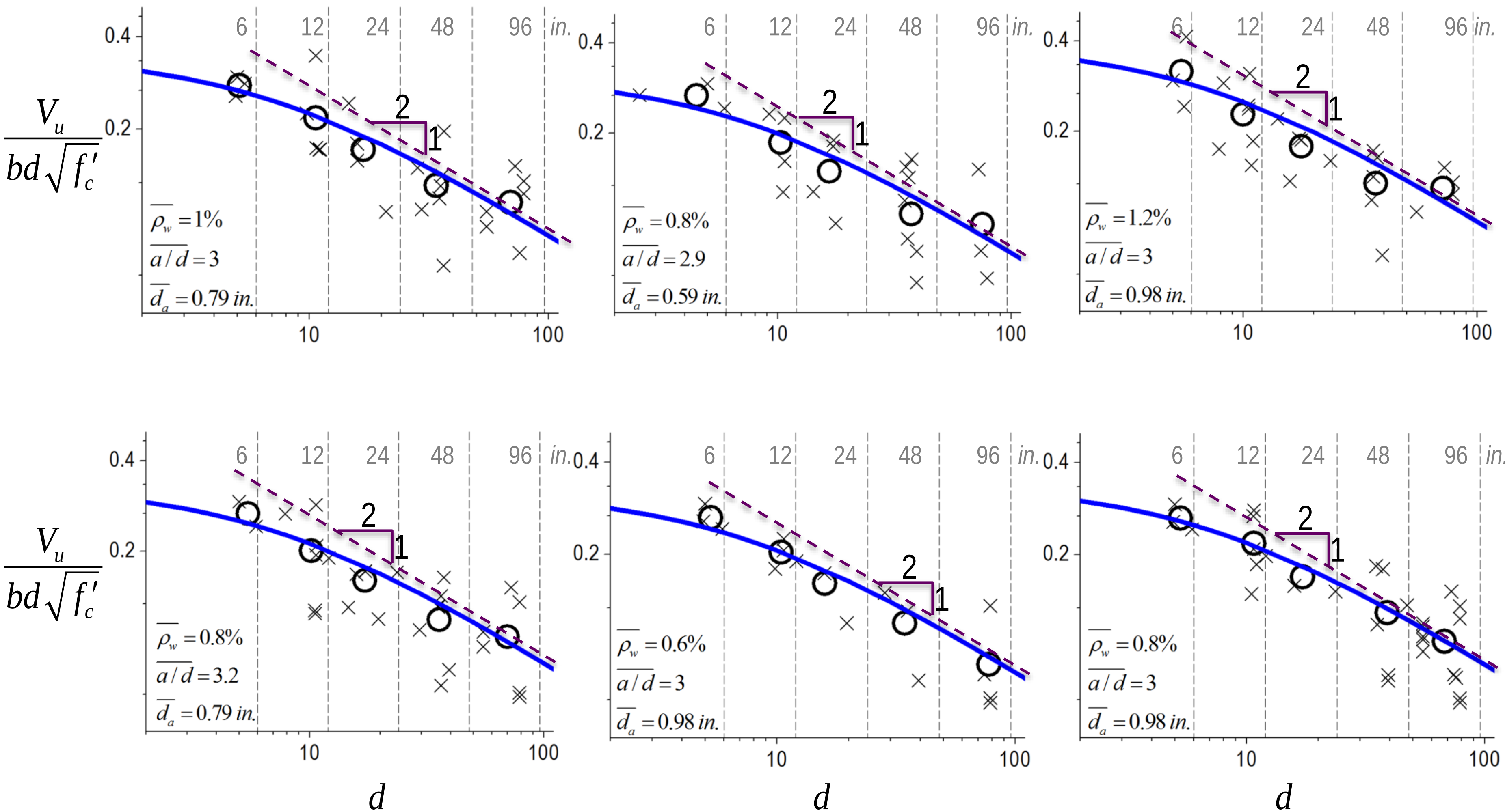
Interval averages of ρ_w, a/d, d_{aggr} vary over size range!



Restricting Strength Range of Data to Achieve Approximately the Same Means of ρ_w, a/d, d_a in All Size Intervals



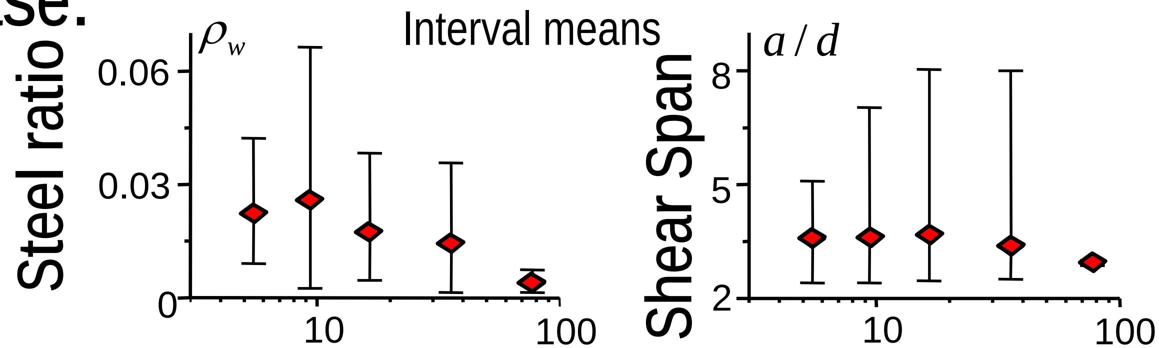
Mean size effects revealed by several data subbases with <u>bias filtered out</u> to make averages of $\rho_{\mathbf{w}}$, a/d, d_{aggr} <u>uniform</u>



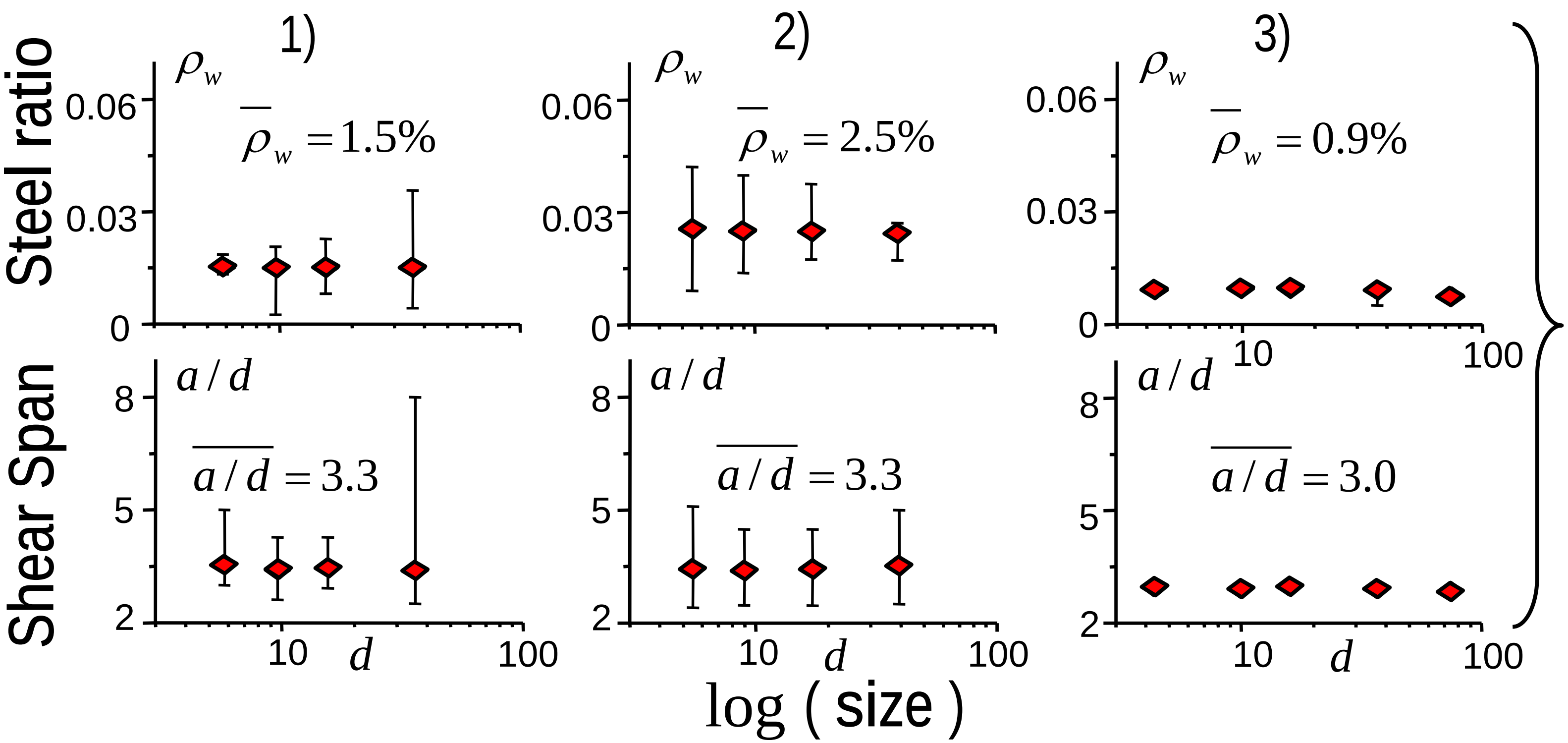
Note: The means agree with ACI-446 size effect curve. The slope is never steeper than -1/2 (thermodynamically impossible anyway).

Database Contaminated by Variation of Secondary Parameters

Entire Database:



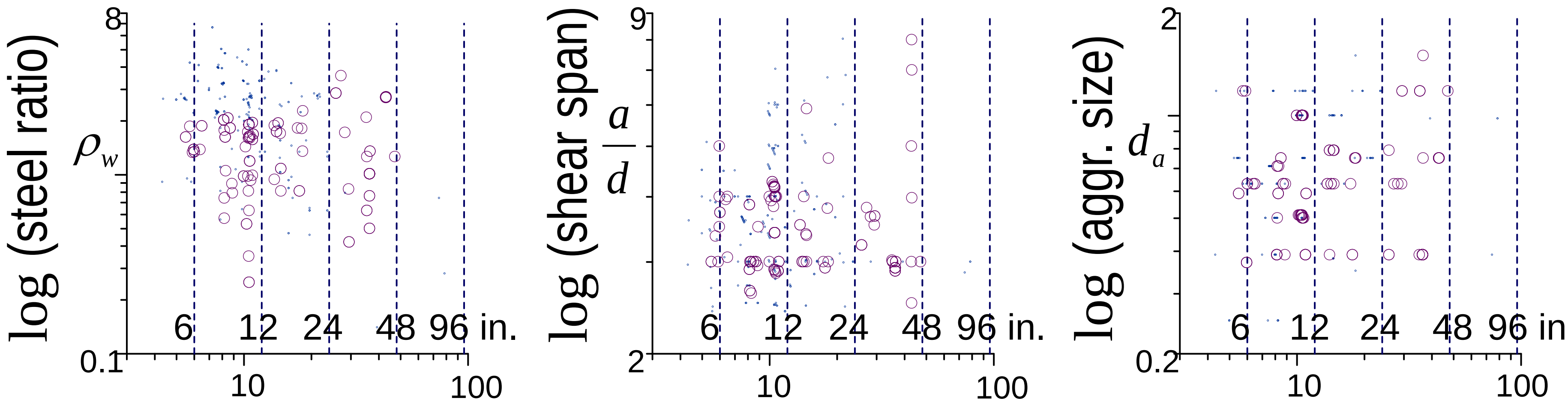
3 Filtered Databases:



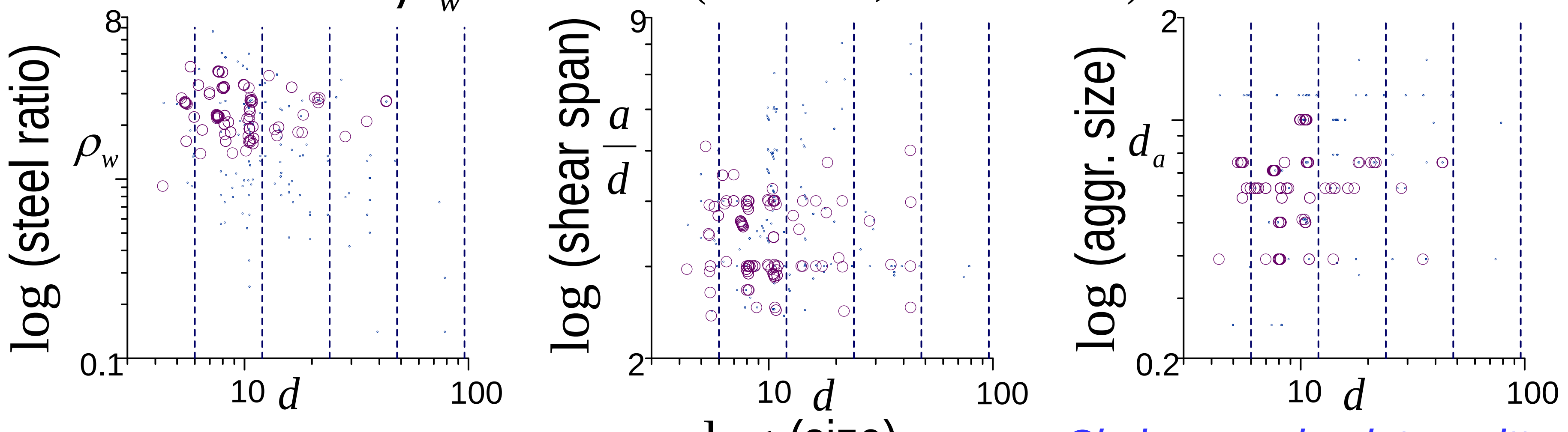
Nearly uniform steel ratio, shear span and aggregate size

Filtered Database with Nearly Uniform Steel Ratio, Shear Span & Aggregate Size

$\overline{\rho}_w = 1.5\%$ $\quad (\overline{a/d} = 3.3, \quad \overline{d}_a = 0.67 \text{ in.})$



$\overline{\rho}_w = 2.5\%$ $\quad (\overline{a/d} = 3.3, \quad \overline{d}_a = 0.67 \text{ in.})$

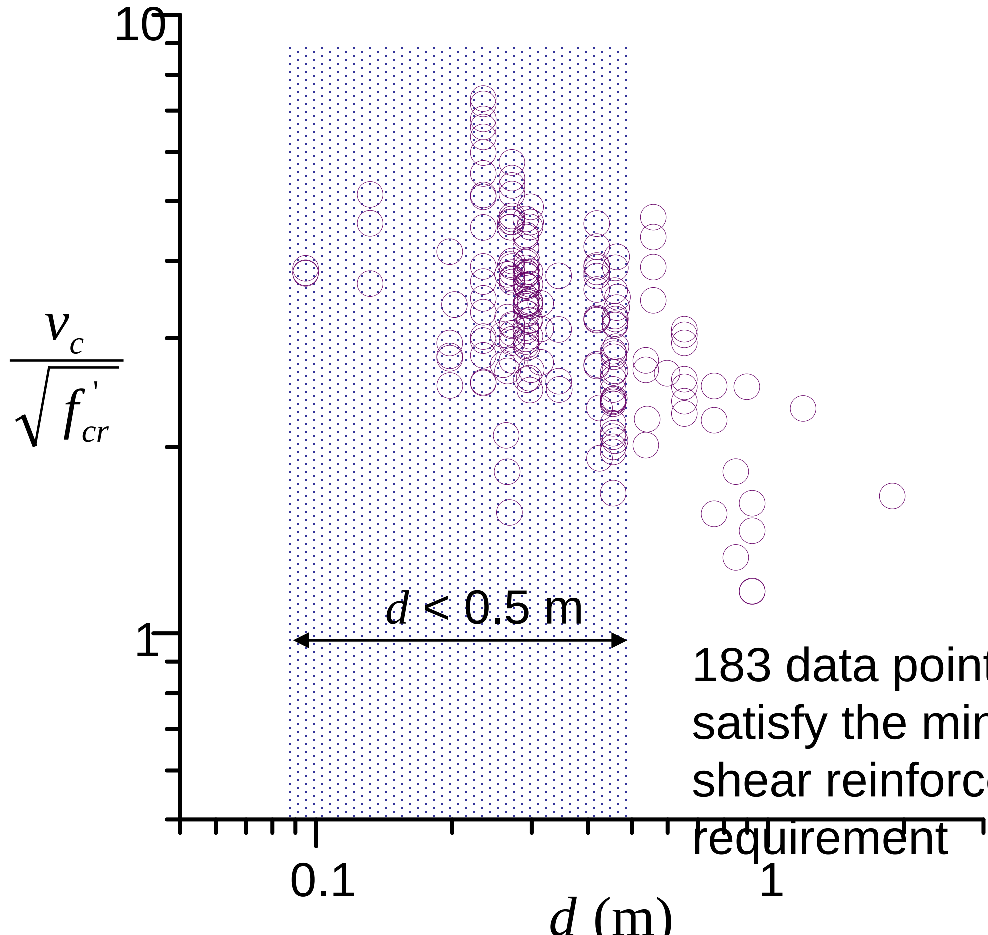


Circles remain, dots omitted

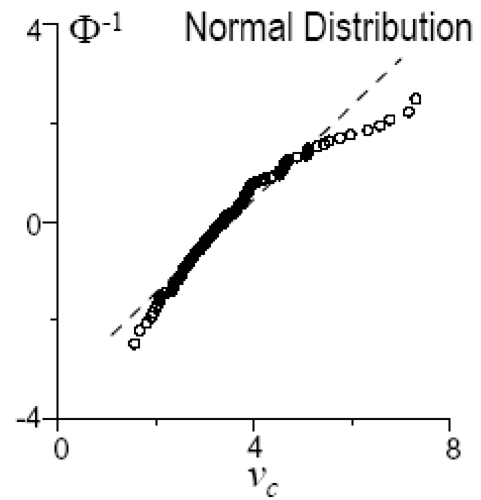
Decrease of Safety Margin with Increasing Size

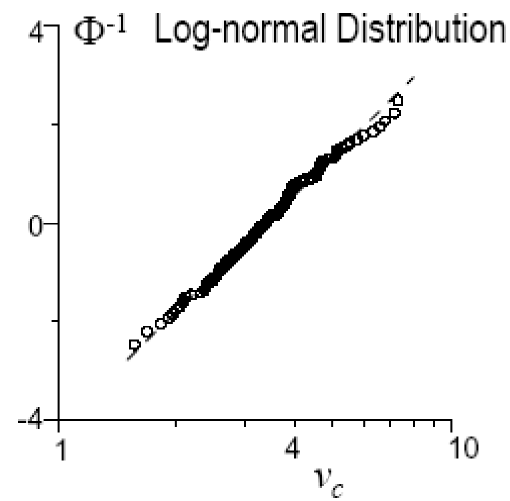
Probability Distribution of Shear Strength

Could we treat entire database as a statistical population? No except for the part of size range where size effect is weak



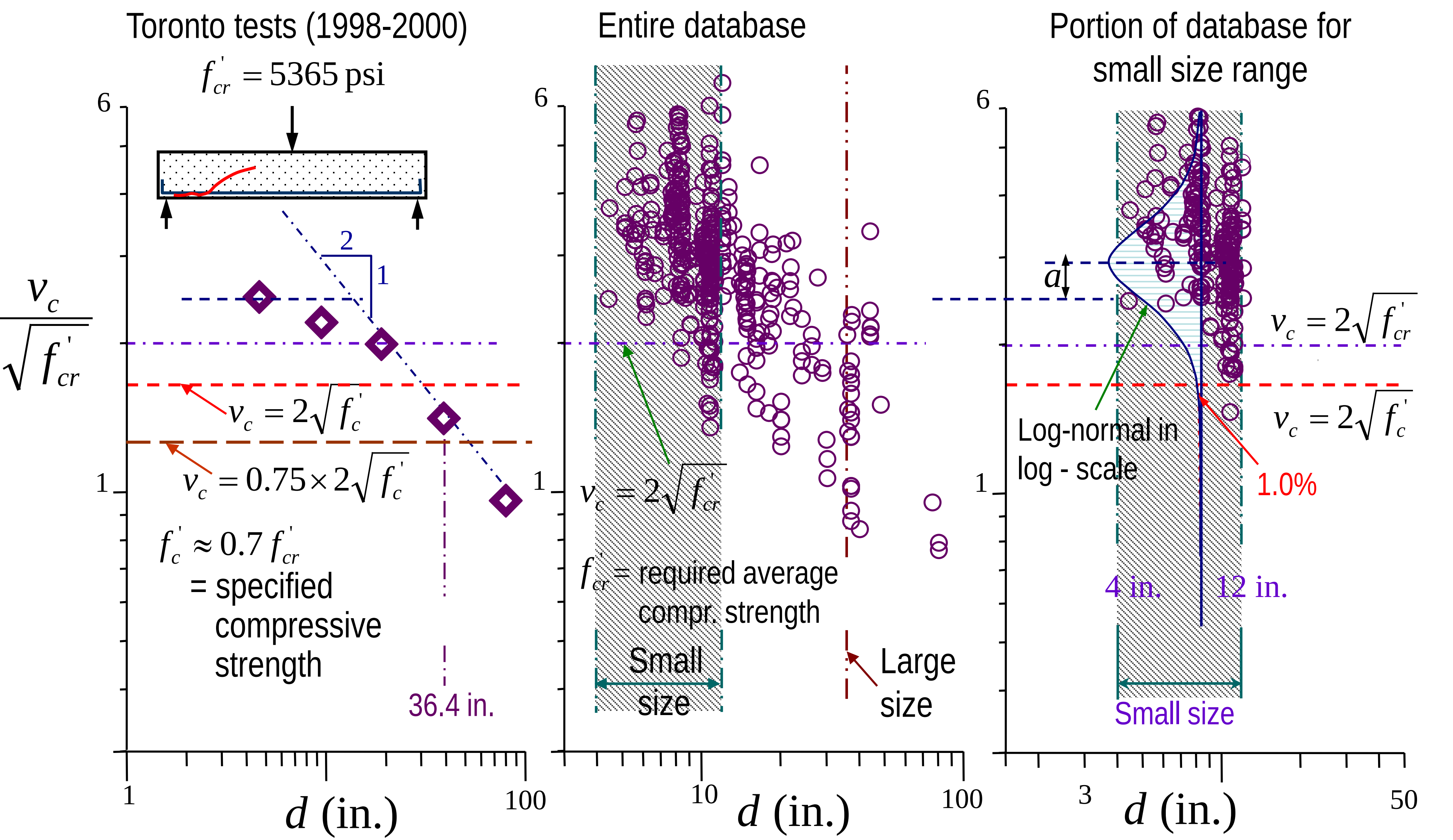
183 data points, all satisfy the minimum shear reinforcement requirement





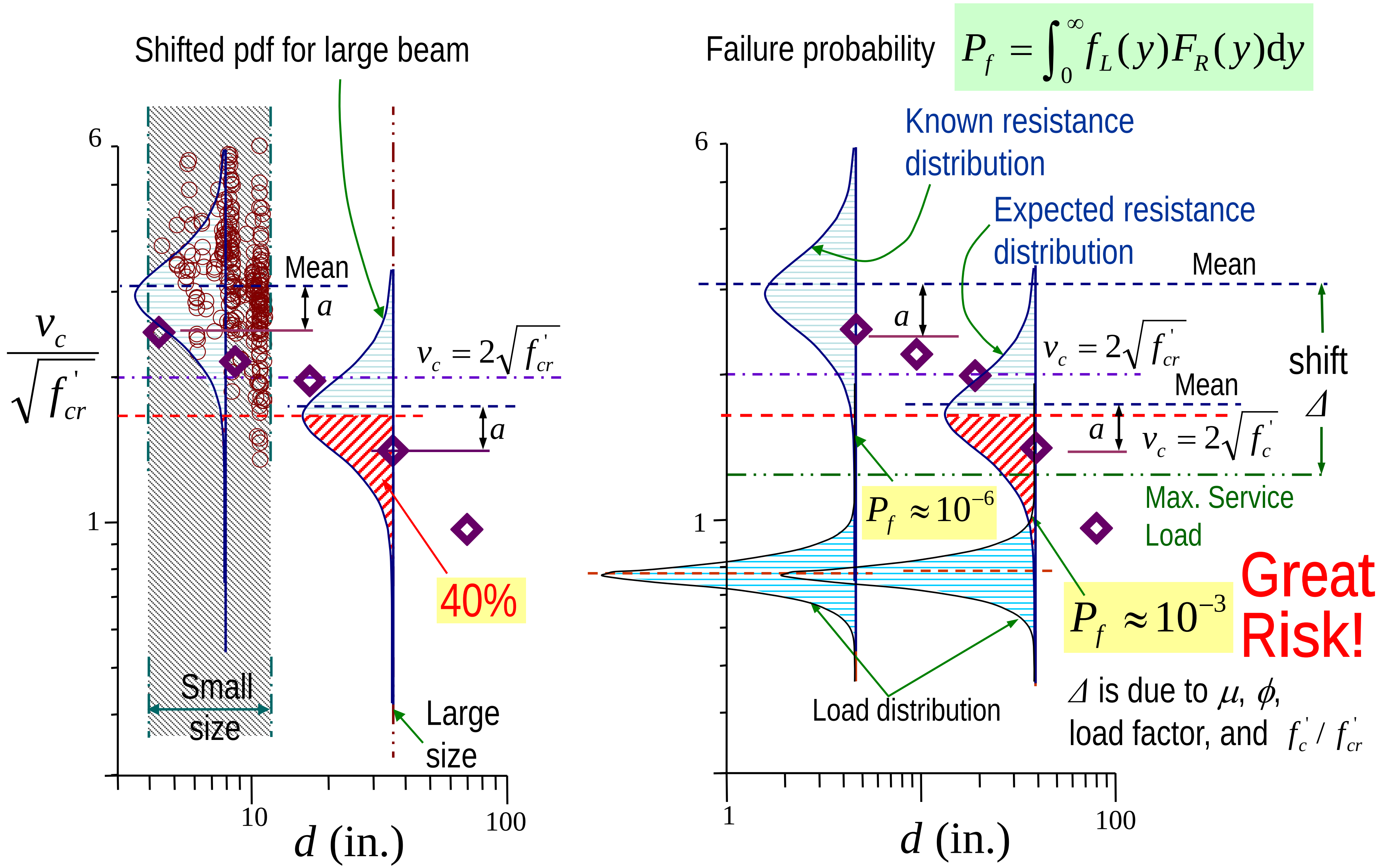
For d < 0.5 m, lognormal distribution is a better description for shear strength because of the scatter insulting from variability of secondary characteristics, e.g. ρ, ρ, a/d

Relation of Size Effect Test Result to pdf of Database



Shift of Resistance cdf Due to Size Increase

(No Stirrups)



Do stirrups suppress the size effect?

Size effect factor for beams with stirrups

$$\vartheta_s = \frac{1}{\sqrt{1 + d / d_{s0}}}$$

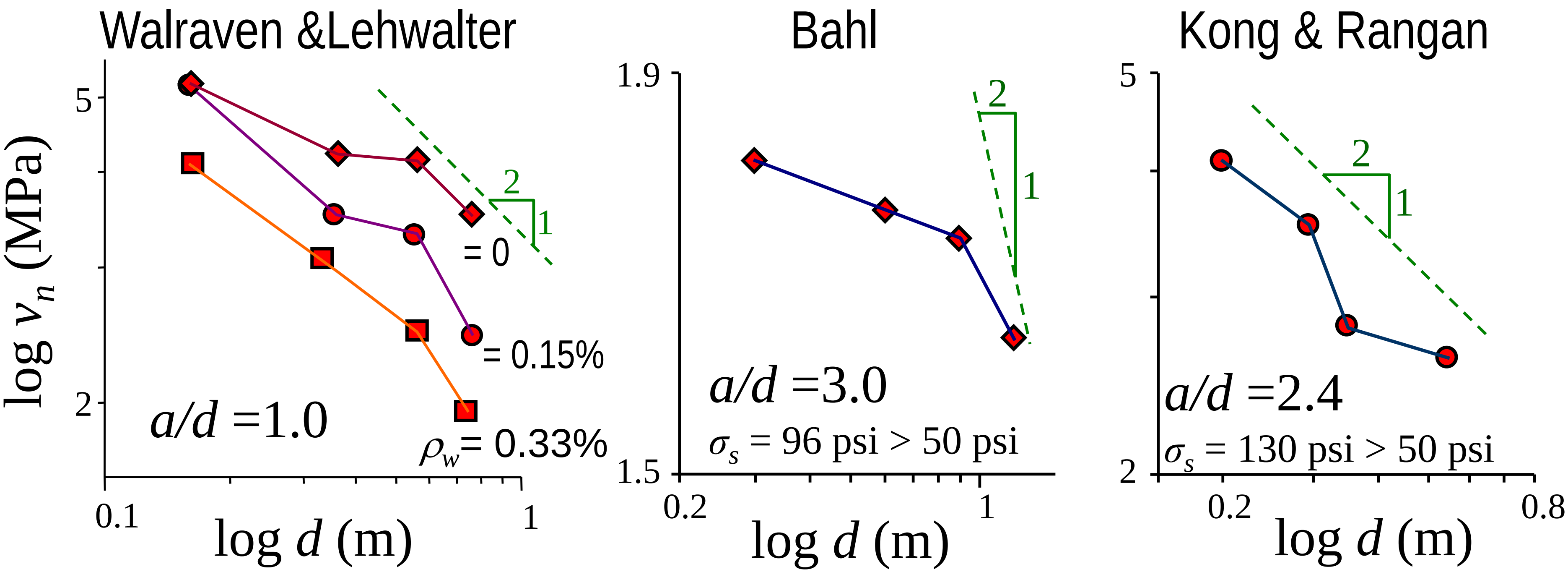
$$d_{s0} = 10 d_0$$

(type 2 size effect)

Classical tests of geometrically similar R.C. beams made with the same concrete indicate that stirrups mitigate but do not eliminate the size effect:

a) Deep beams

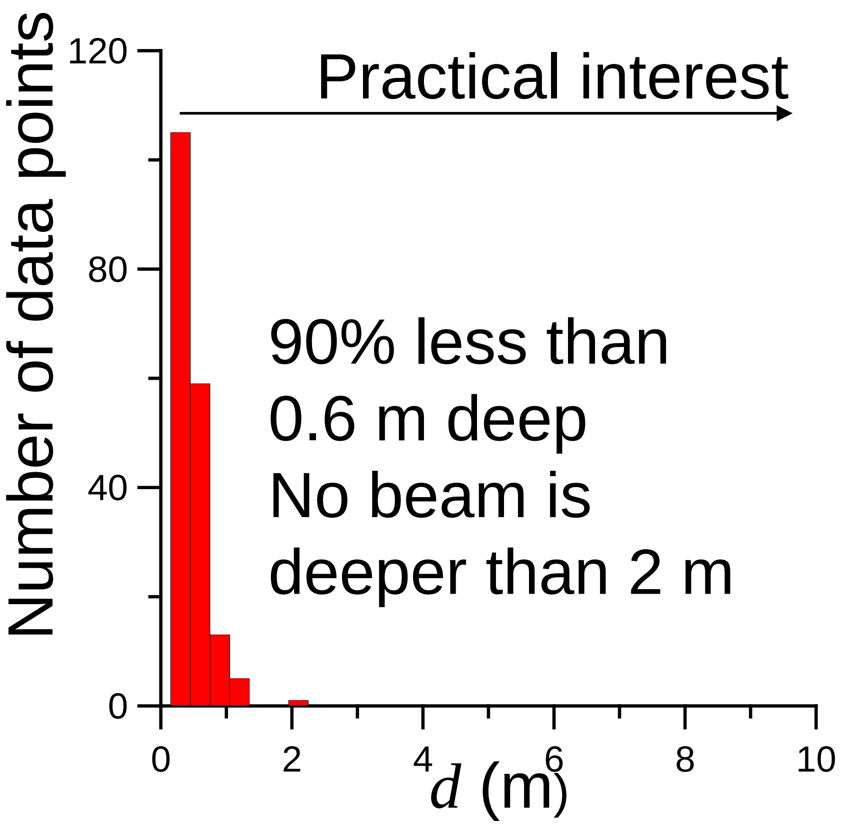
b) Slender beams

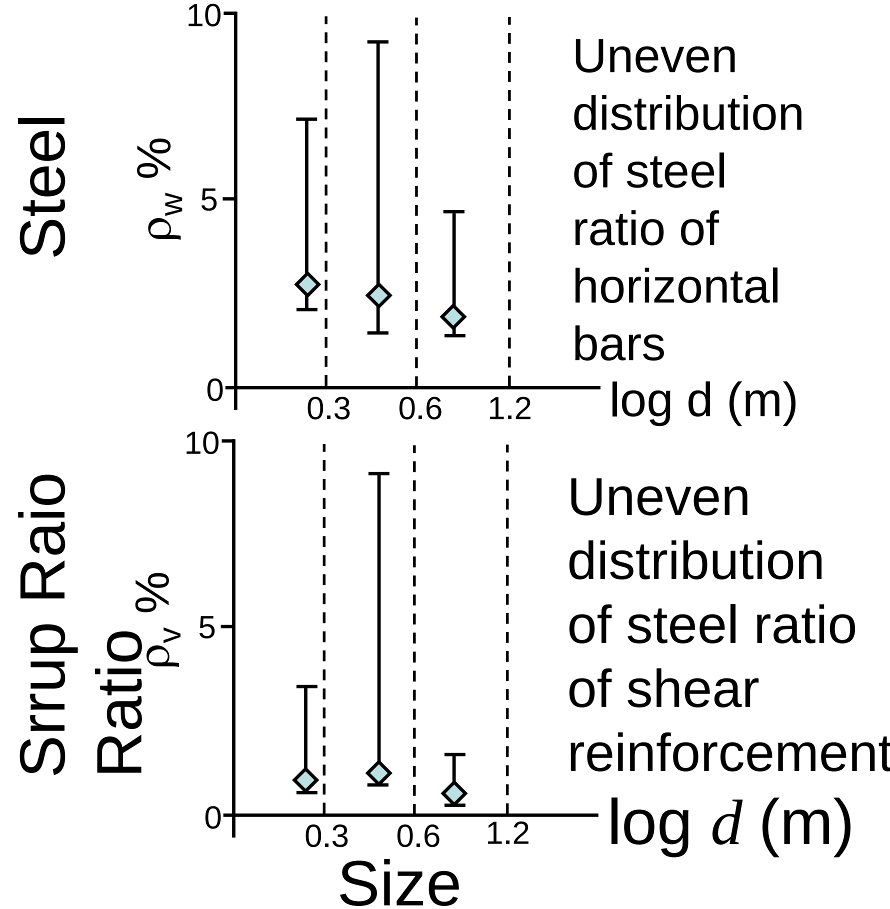


Statistical Bias in Database

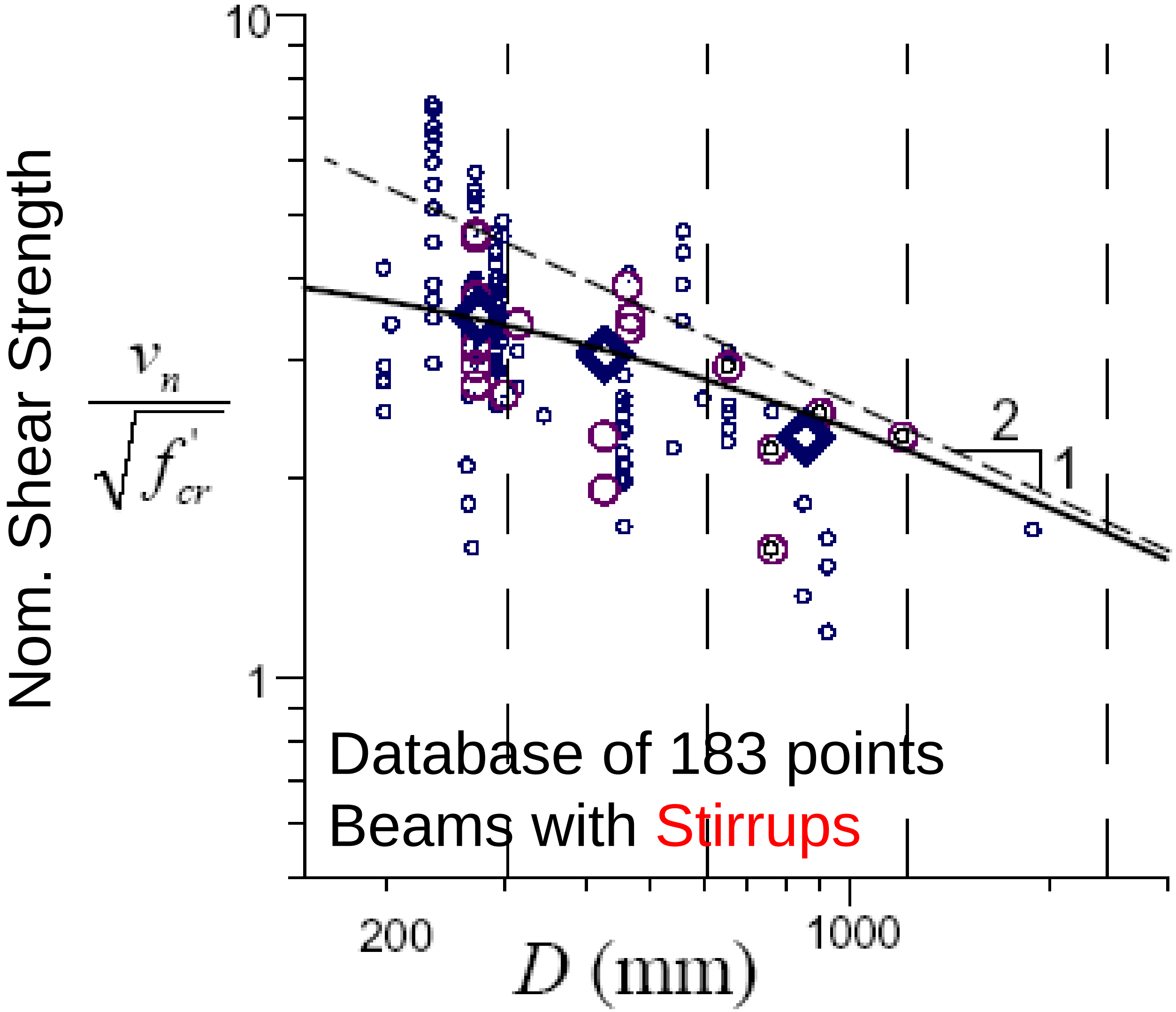
Two main causes of statistical bias:

1) The data points are ***crowded*** in small size range
2) In different size ranges, the distributions of ***secondary influencing parameters are not uniform***





Regression After Suppressing the Bias



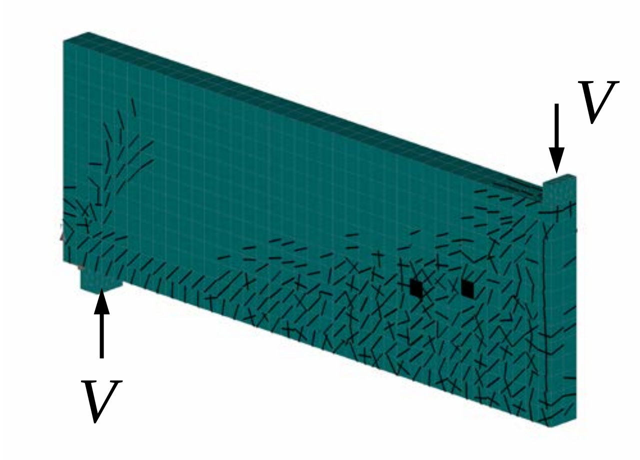
Size

Mean ρ_w = 1.9%
Mean a/d = 3.3
Mean v_s = 0.6 Mpa
Mean d_a = 20 mm

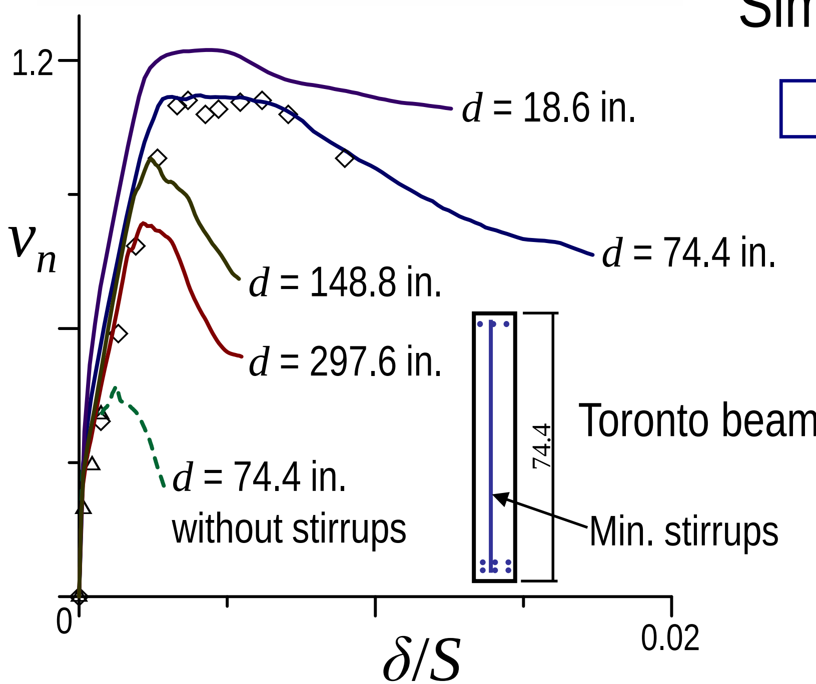
Tiny circles: points filtered out
Big circles: points retained
Blue diamonds: interval centroid

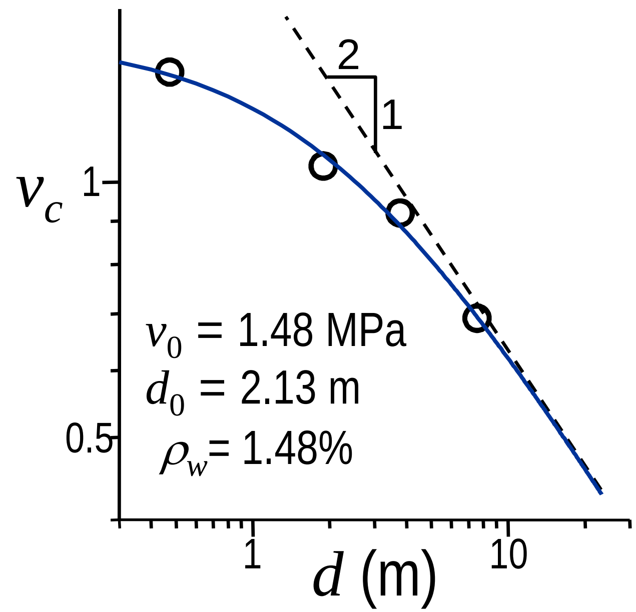
Numerical Simulations Based on M4 Model

Calibrated by Toronto Tests



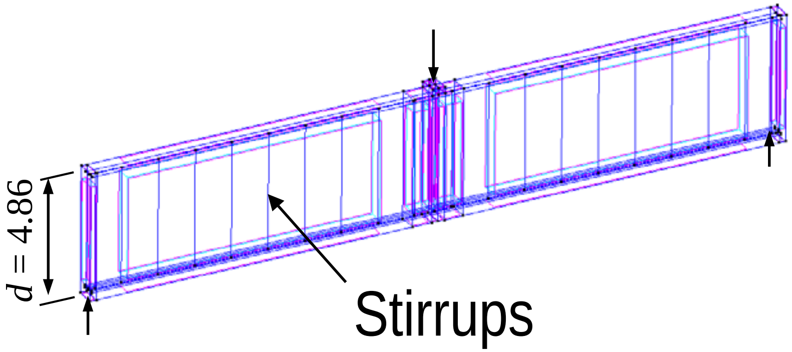
Simulations

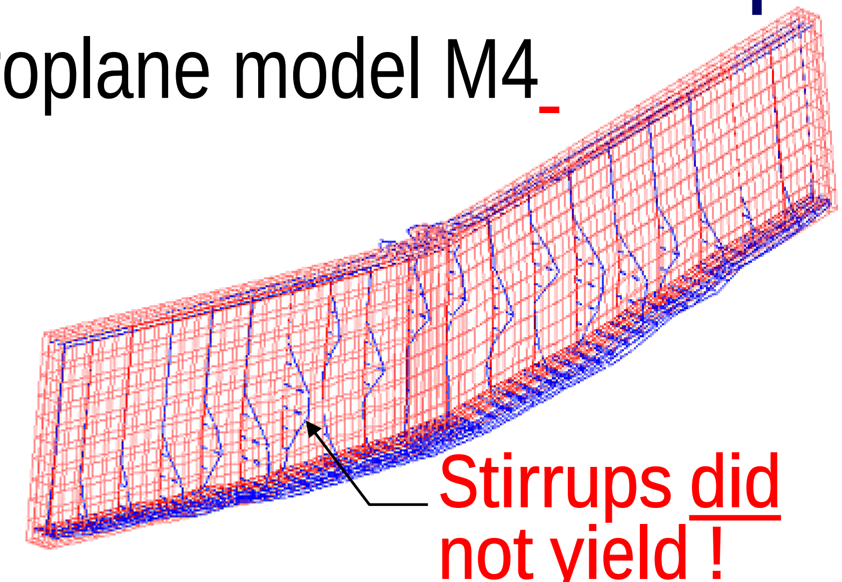


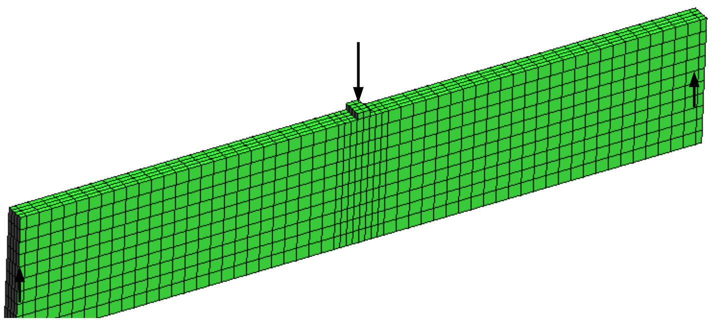


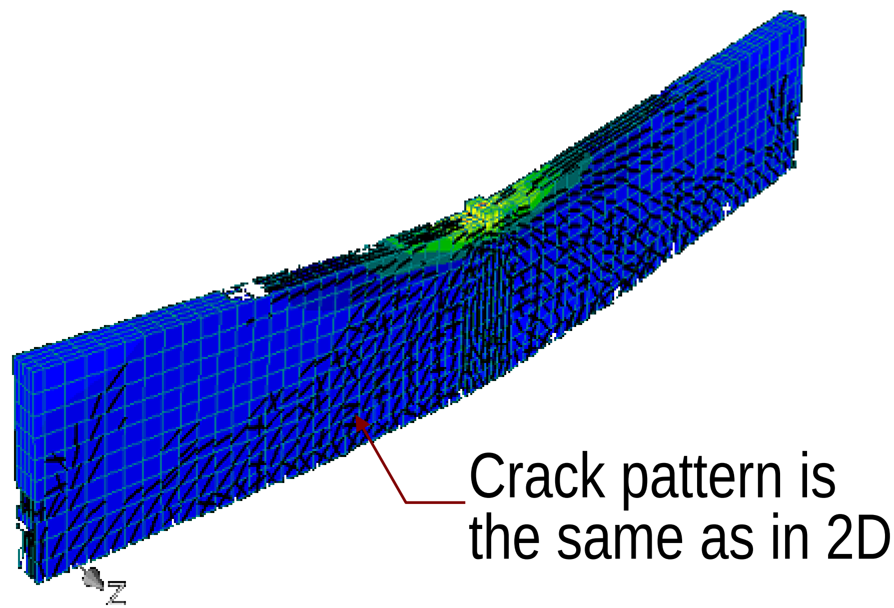
3D FEM simulation of beams with stirrups

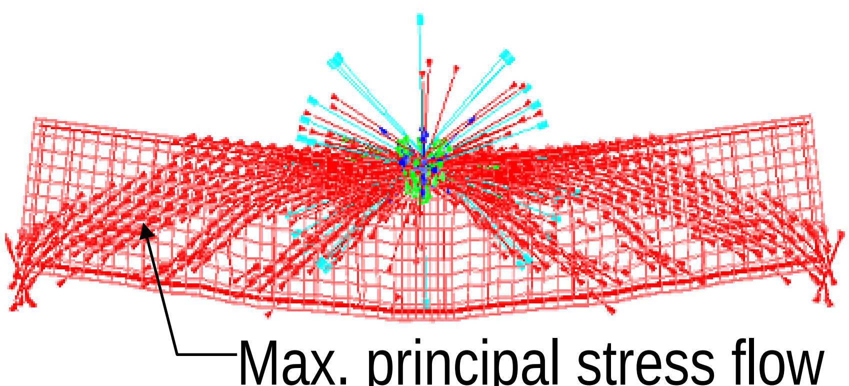
Beam depth d = 4.86m. Microplane model M4



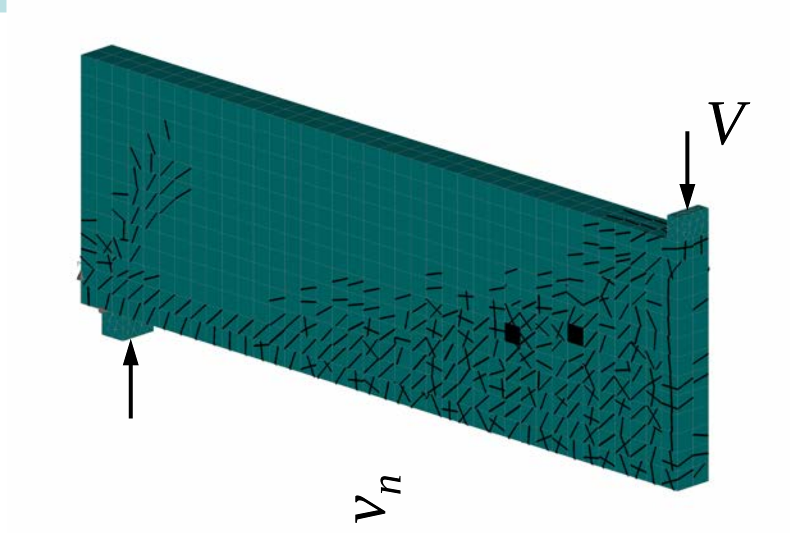


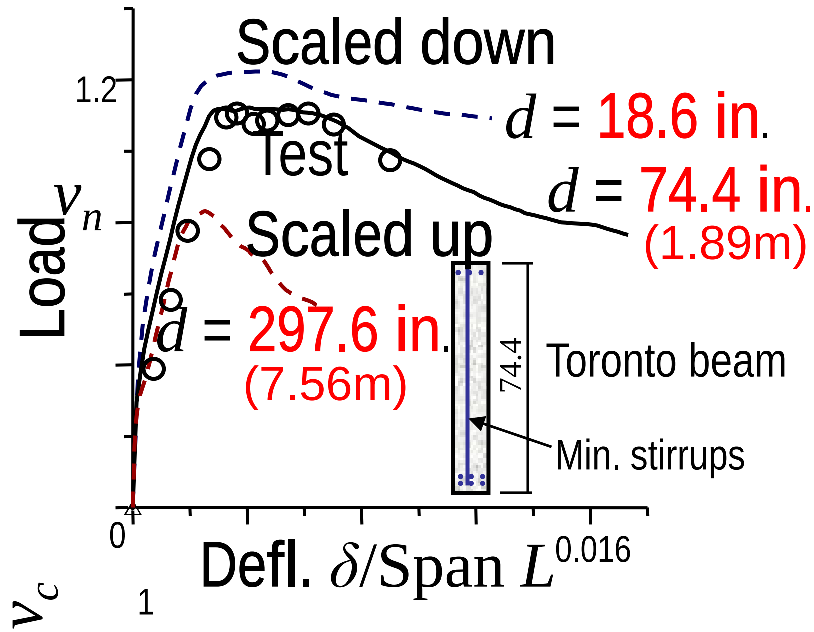




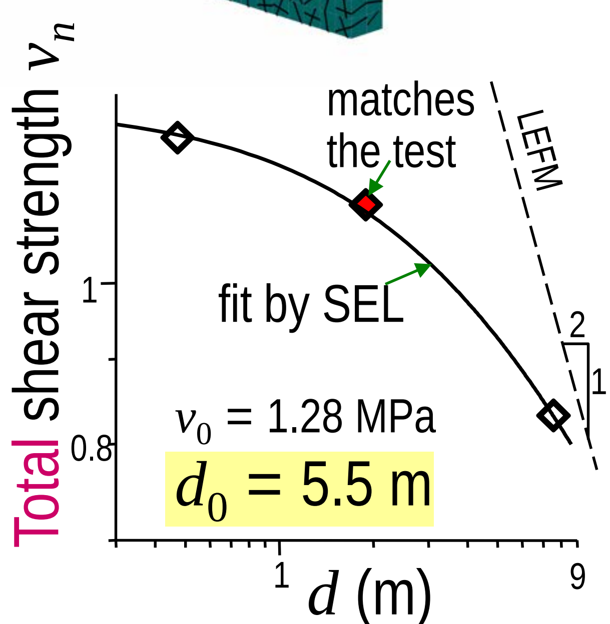


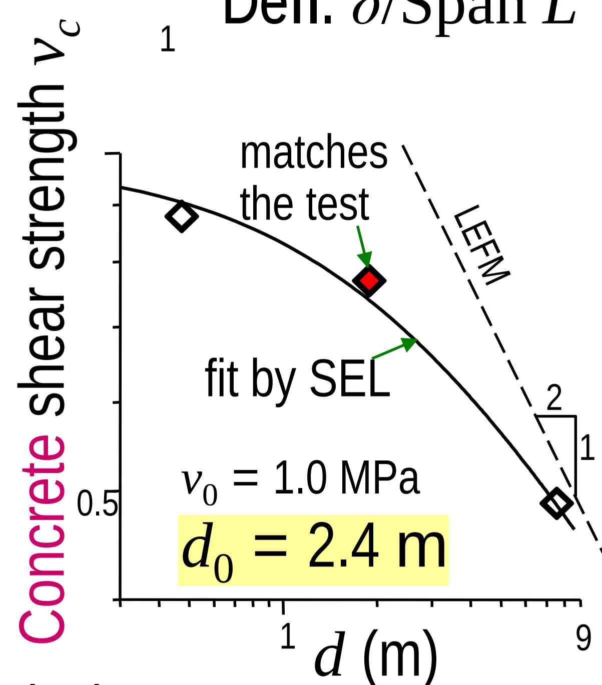
Crack-Band Microplane FEM Simulations of 1.89 m Deep Toronto Beam, Min. Stirrups





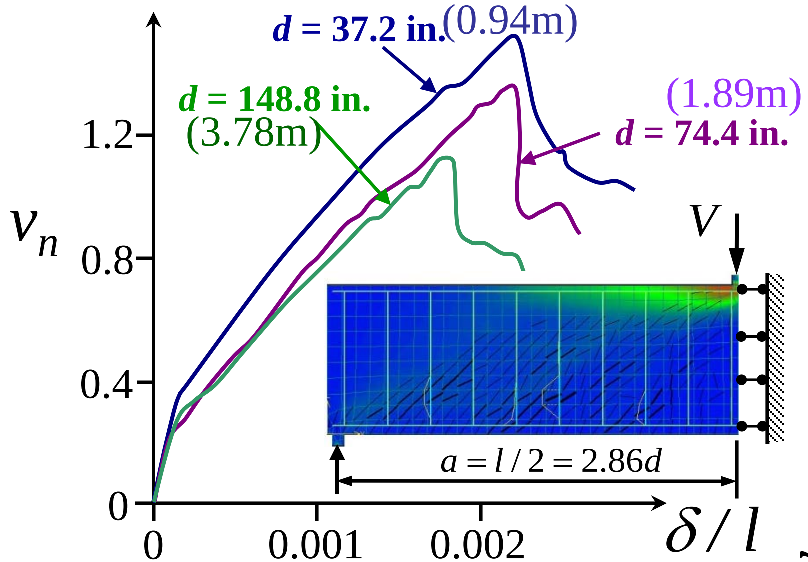
Much Deeper Beam

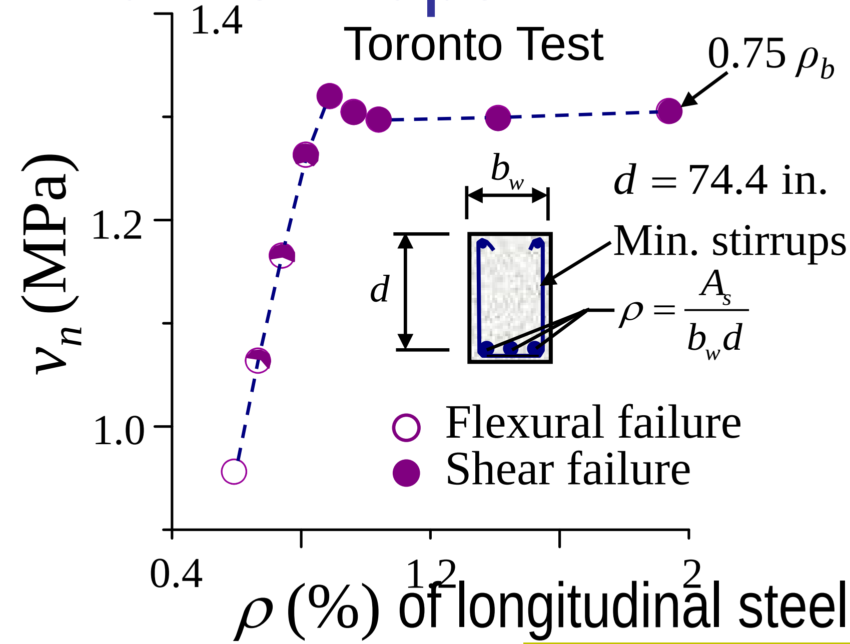


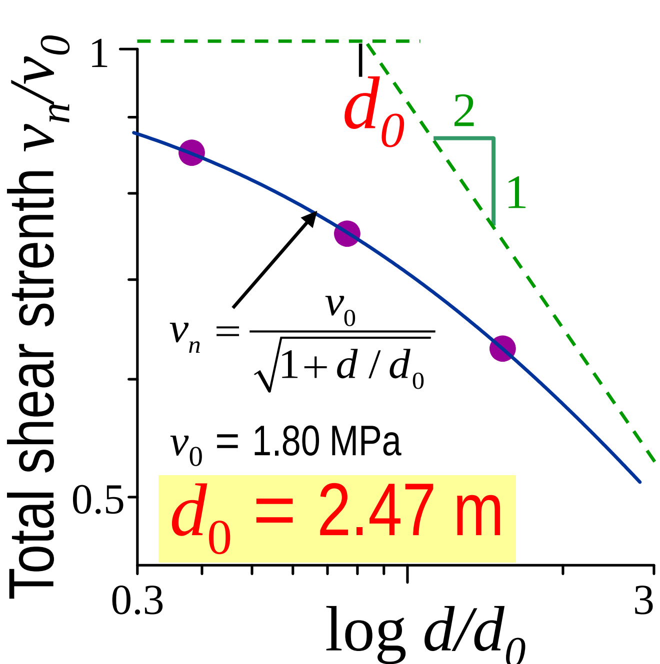


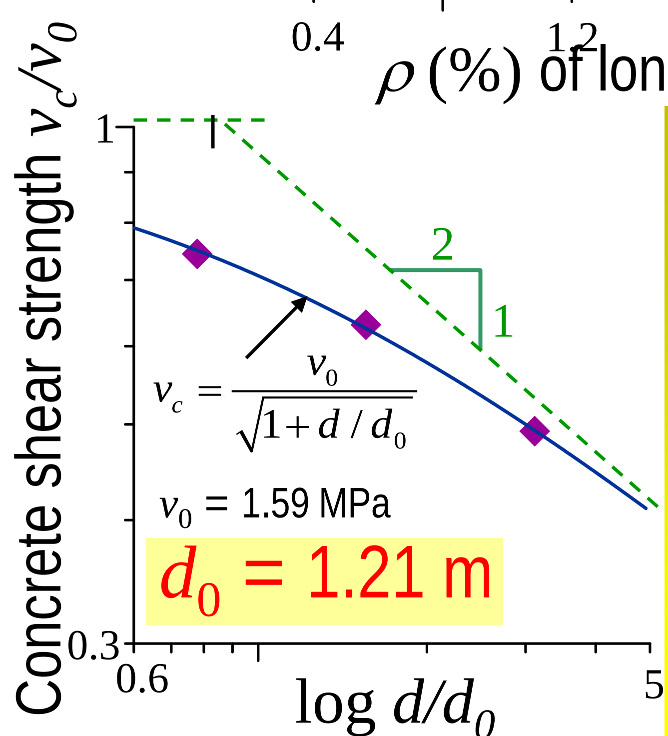
log (size)

Crack-Band Microplane FEM Simulations of Shear of Beams with Minimum Stirrups





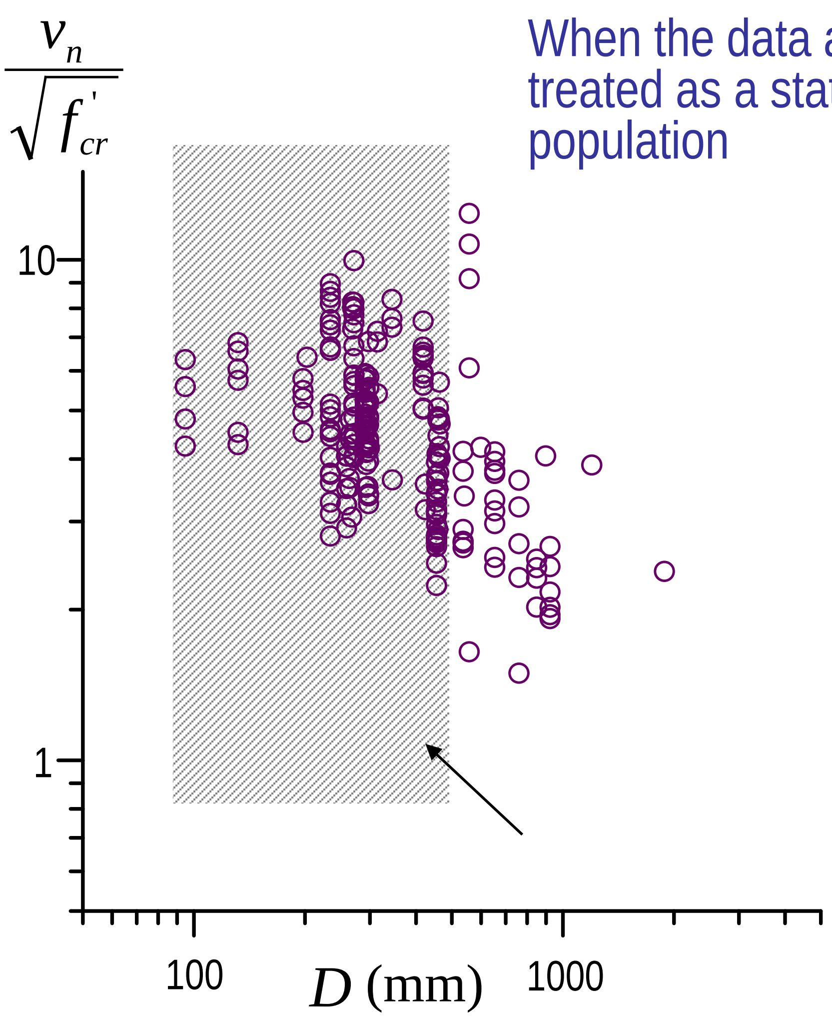




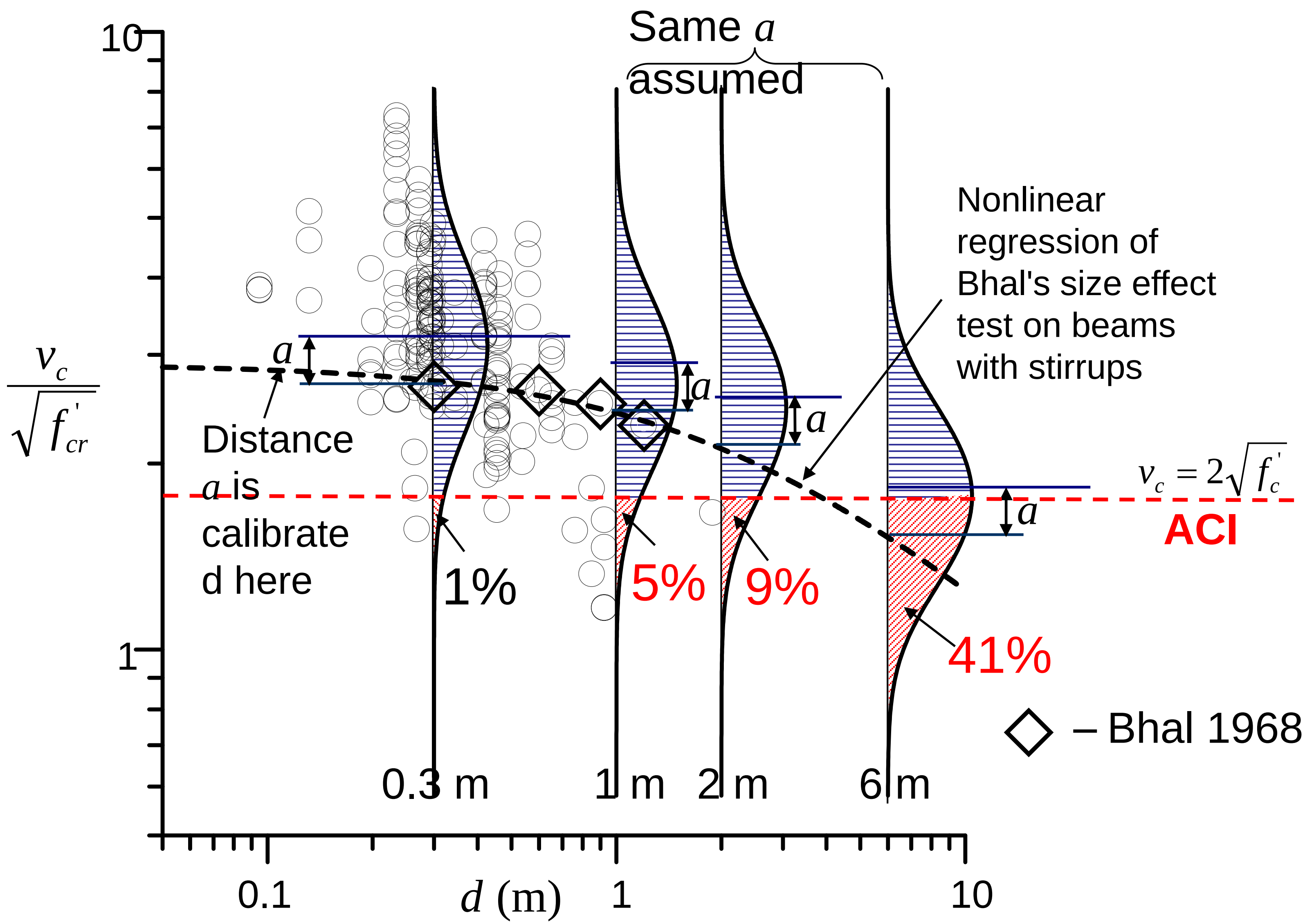
COMPARE:
For no stirrups,
d_0 = 0.27 m.
So, the stirrups push the size effect up by ~ 1 order of magnitude.

Newly Collected Database for Shear Beams with Stirrups

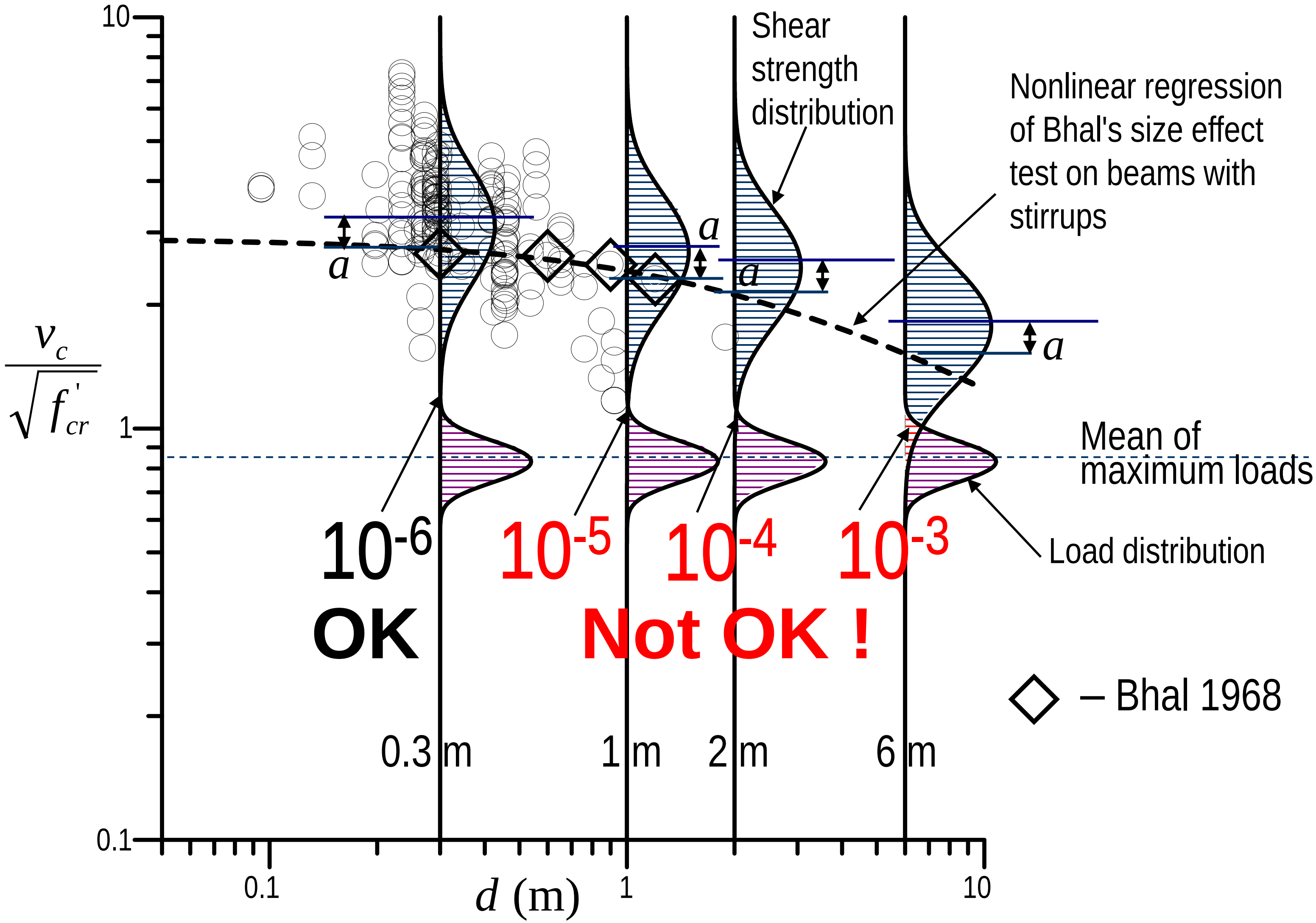
When the data are treated as a statistical population



Unequal Safety Margin if Size Effect Ignored



Failure Probability of RC Beams with Stirrups

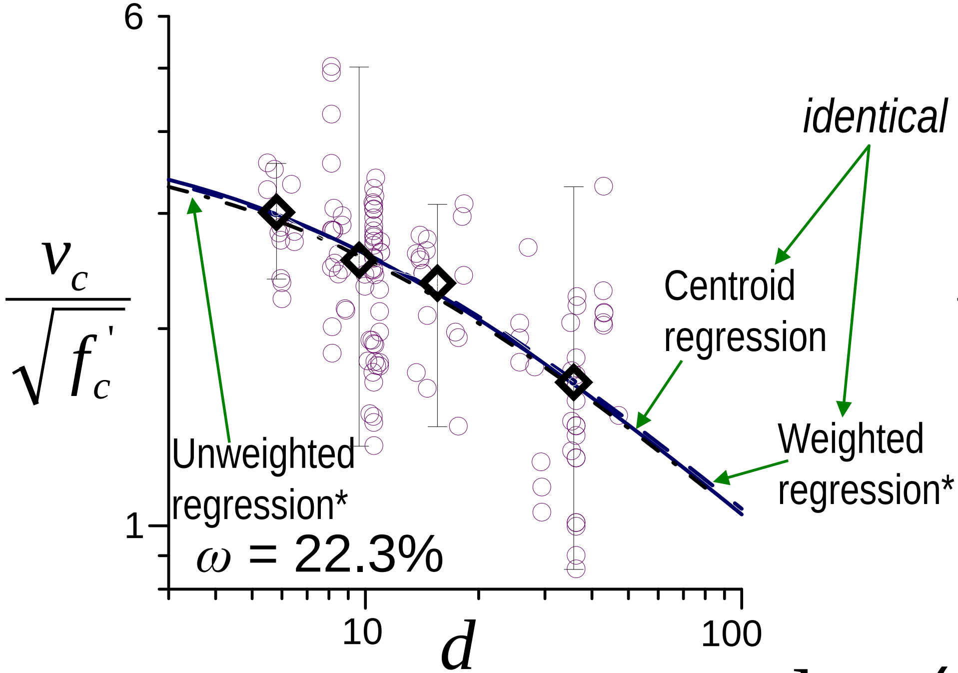


Size Effect Law Fitting Centroids of Filtered Database with Uniform $\overline{\rho}_w, \overline{a/d}, \overline{d}_a$

(purely statistical inference)

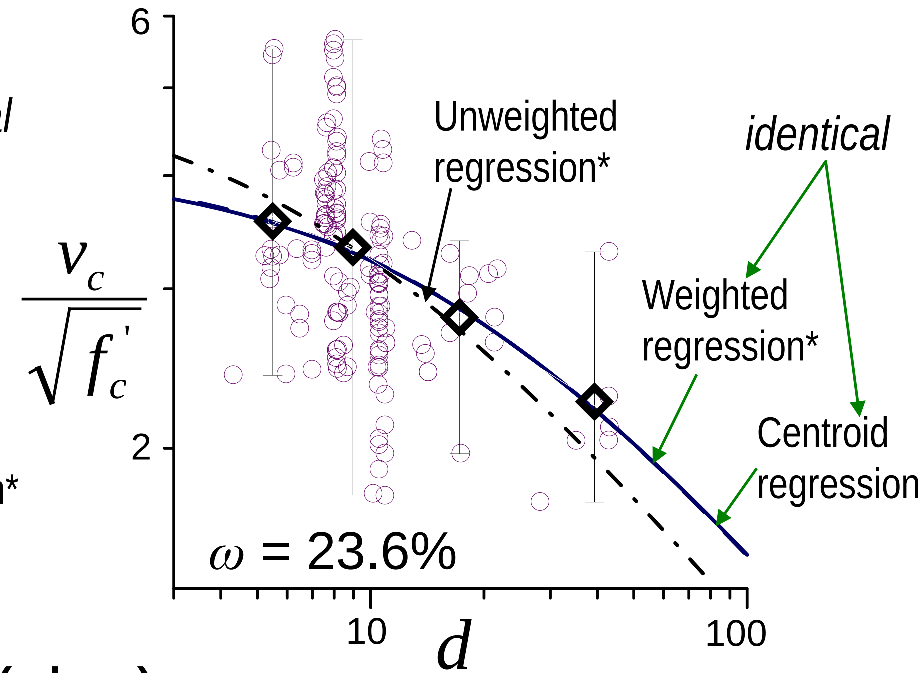
$\overline{\rho}_w = 1.5\%$

$\overline{a/d} = 3.3, \ \overline{d}_a = 0.67$ in.



$\overline{\rho}_w = 2.5\%$

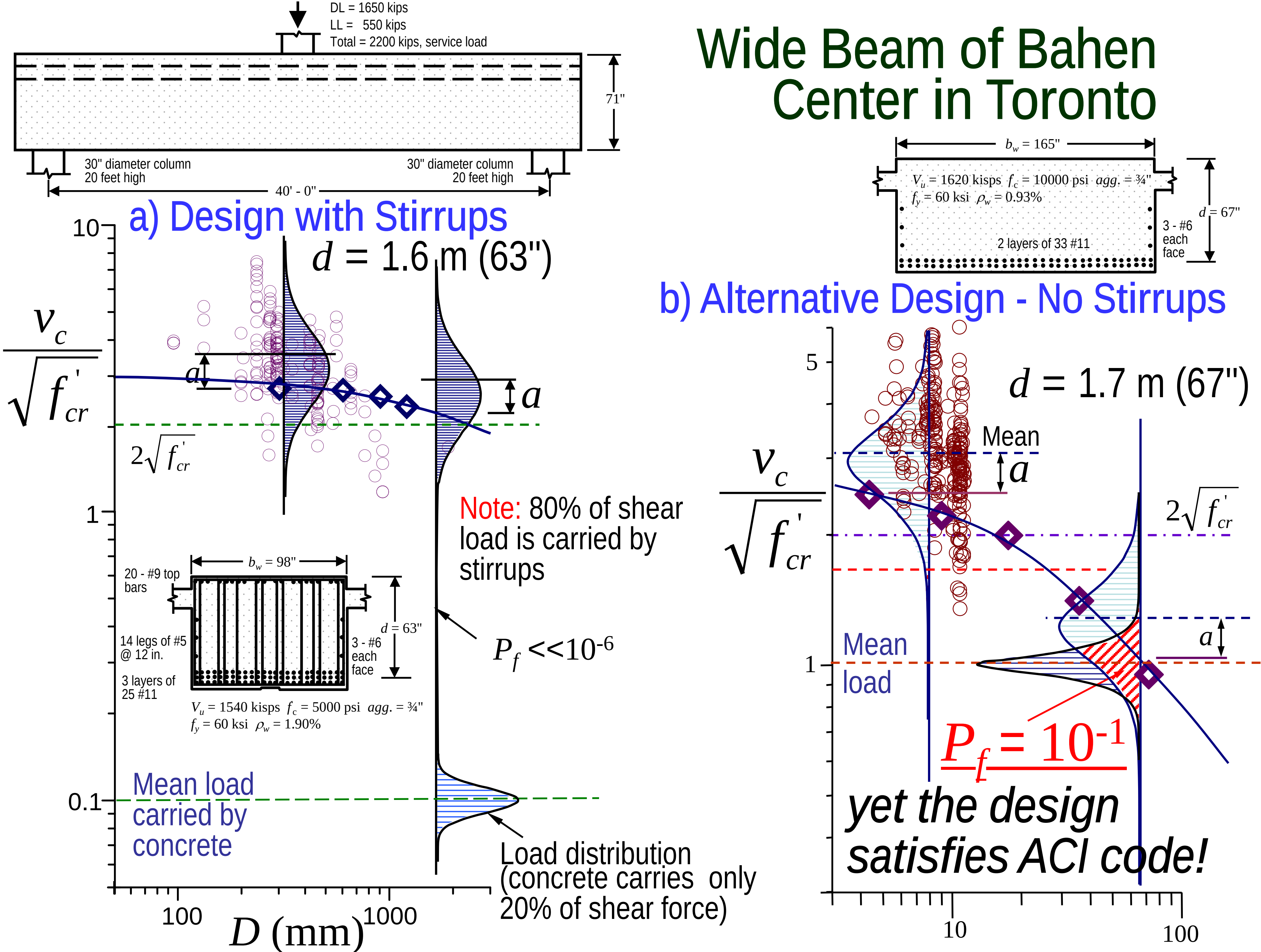
$\overline{a/d} = 3.3, \ \overline{d}_a = 0.67$ in.



$\frac{v_c}{\sqrt{f_c'}}$ vs. d — log (size)

* of individual data points: (weight) ~ 1 / (number of points in interval)

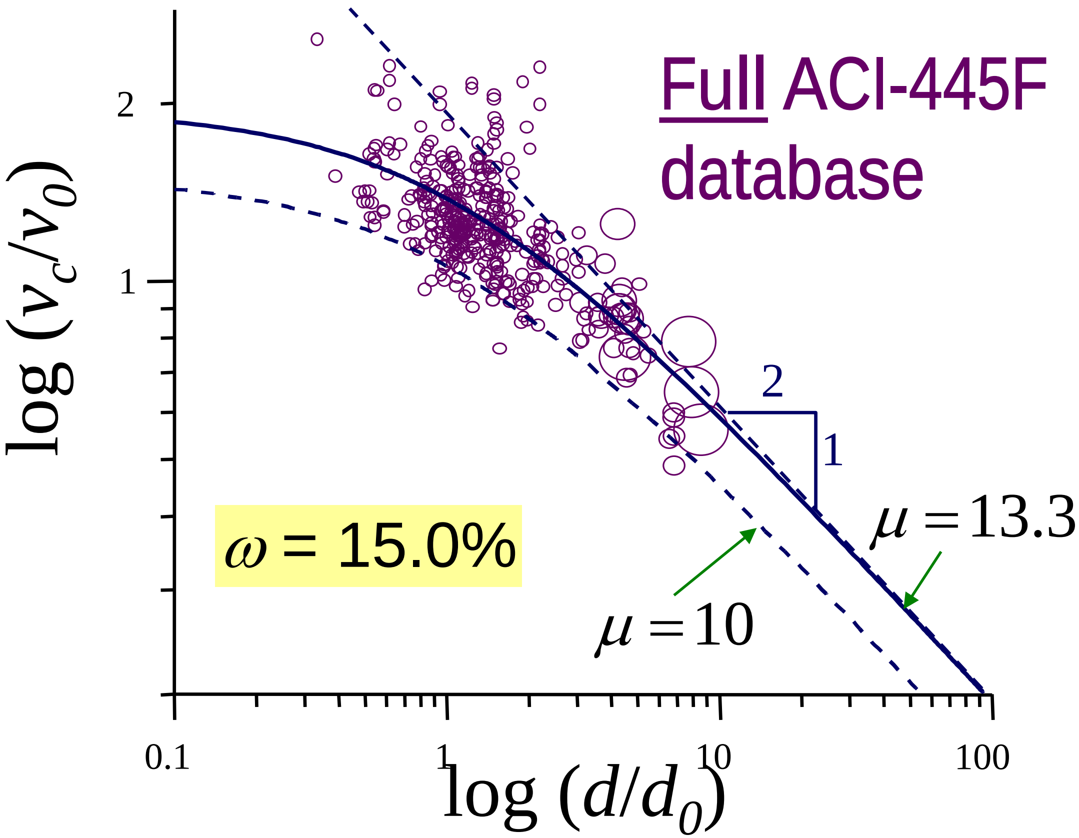
Wide Beam of Bahen Center in Toronto

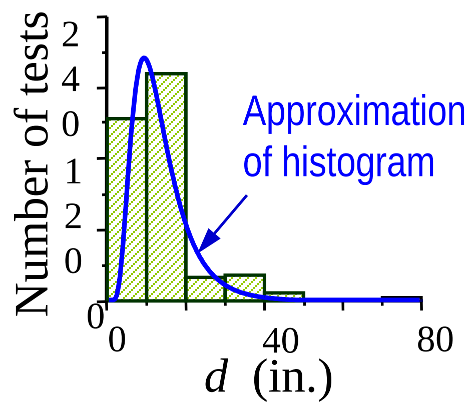


Shear Strength Design Equation Calibrated by Least-Square Nonlinear Regression—ACI-446 Proposal to ACI-318, Aug.2006

$$v_c = \mu \rho_w^{3/8} \left(1 + \frac{d}{a}\right) \sqrt{\frac{f_c^{'}}{1 + d/d_0}} \qquad d_0 = \kappa \ f_c^{'-2/3}$$

$$\kappa = 3800\sqrt{d_a} \text{ if } d_a \text{ is known}, \quad \kappa = 3330 \text{ if not}$$



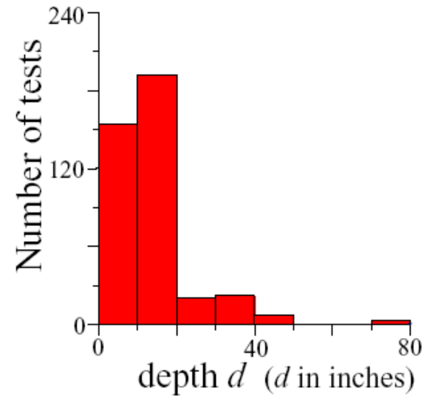


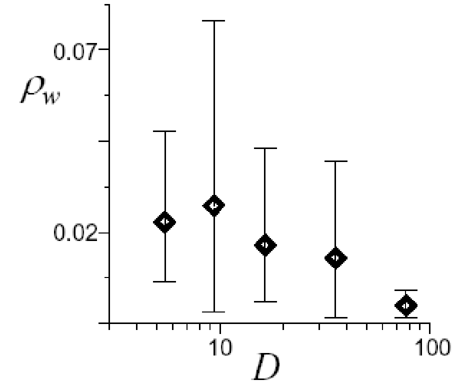
Data points are not uniformly distributed in different size intervals

Capture Size Effect by Proper Statistical Analysis

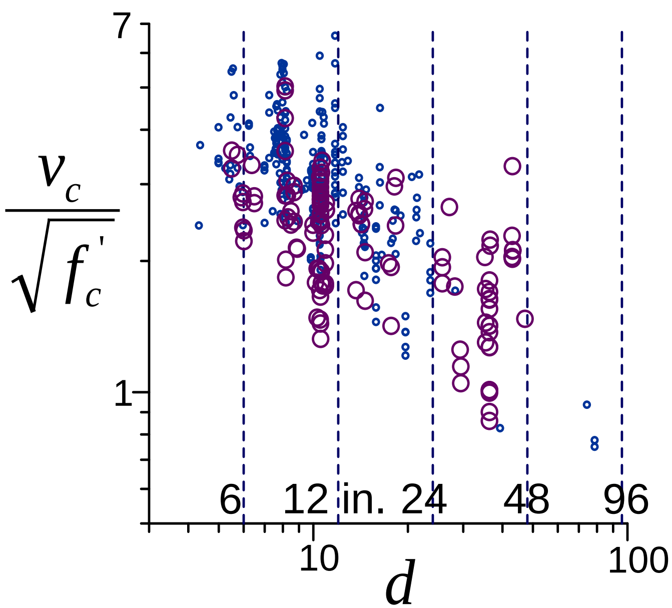
Two main causes of statistical bias:

1) The data points are crowded in small size range
2) In different size ranges, the distributions of secondary influencing parameters are not uniform





Data that remain after filtering



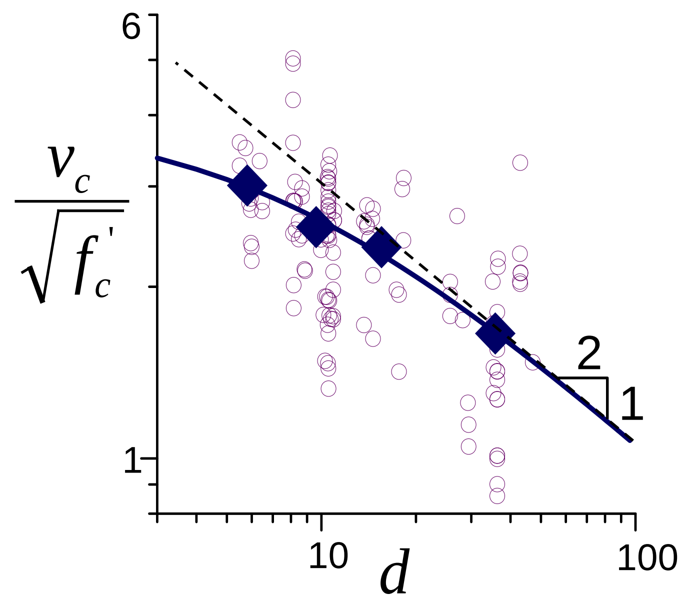
Mean in intervals

$\overline{\rho}_w = 1.5\%$

$\overline{a/d} = 3.3$

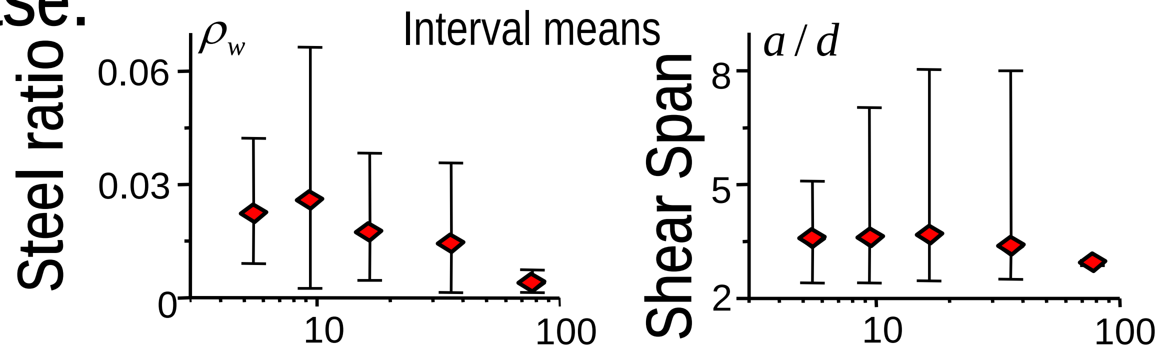
$\overline{d}_a = 0.67$ in.

Regression of Centroids

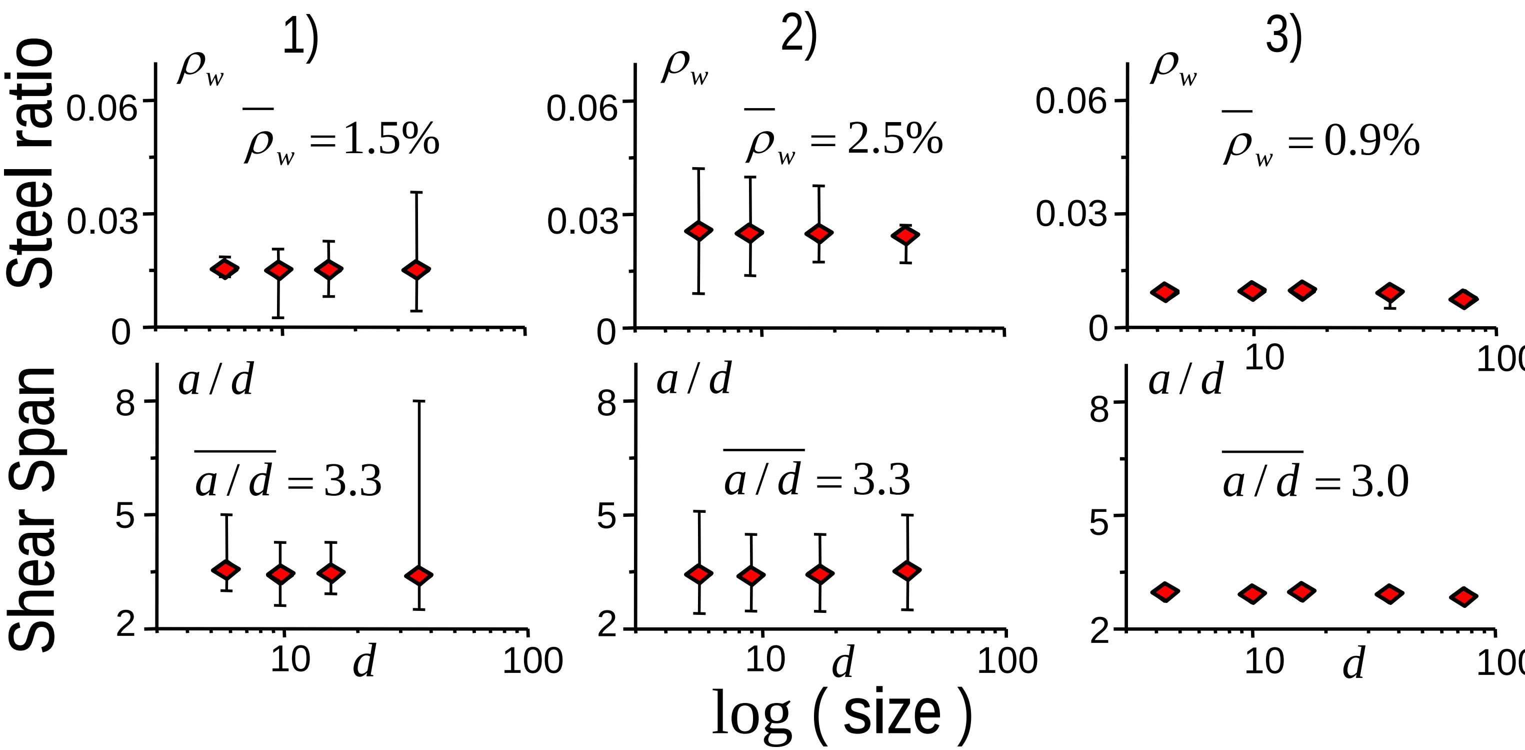


Database Contaminated by Variation of Secondary Parameters

Entire Database:



3 Filtered Databases:

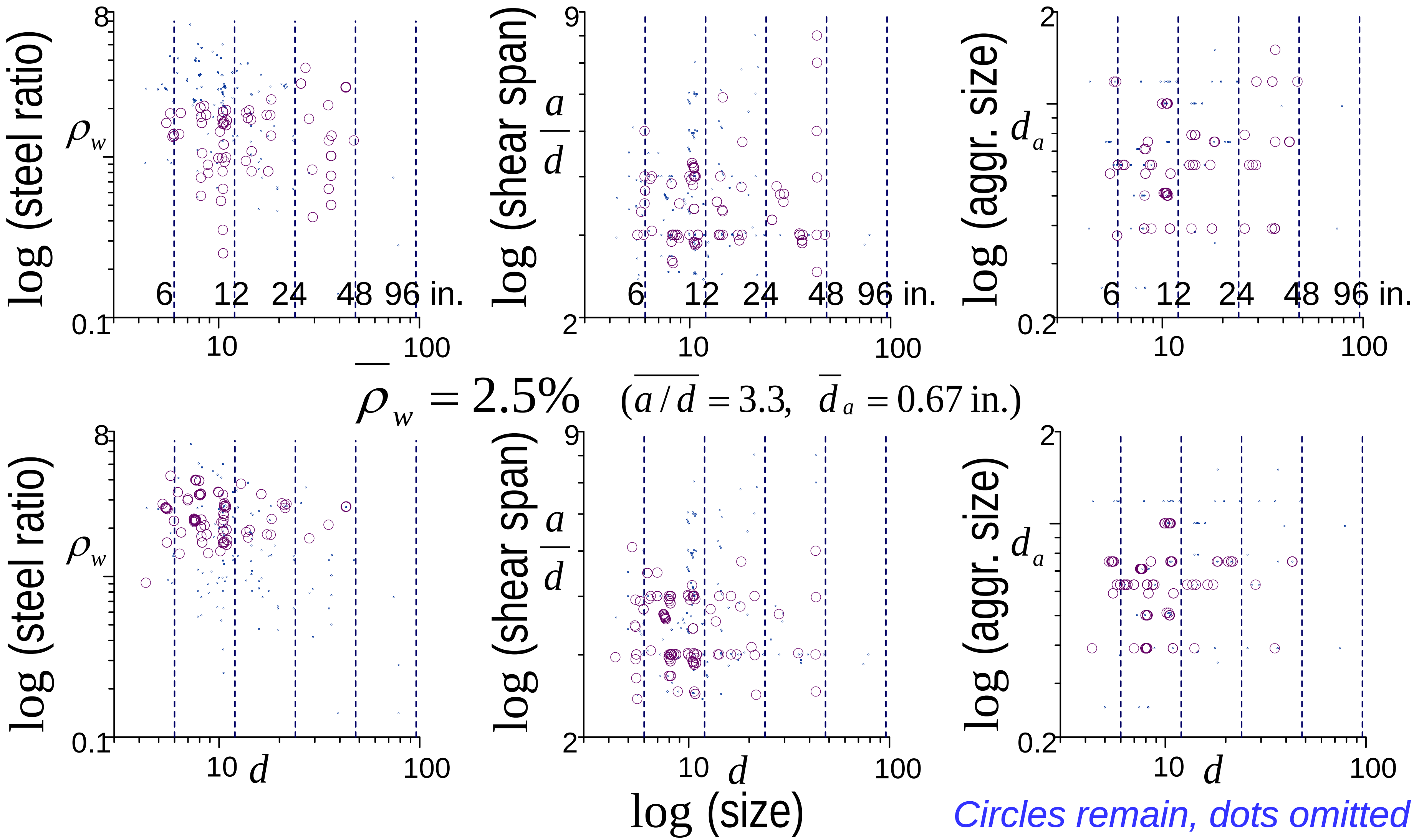


Achieved: Nearly *uniform* *steel ratio, shear span and aggregate size*

Filtered Database with Nearly Uniform Steel Ratio, Shear Span & Aggregate Size

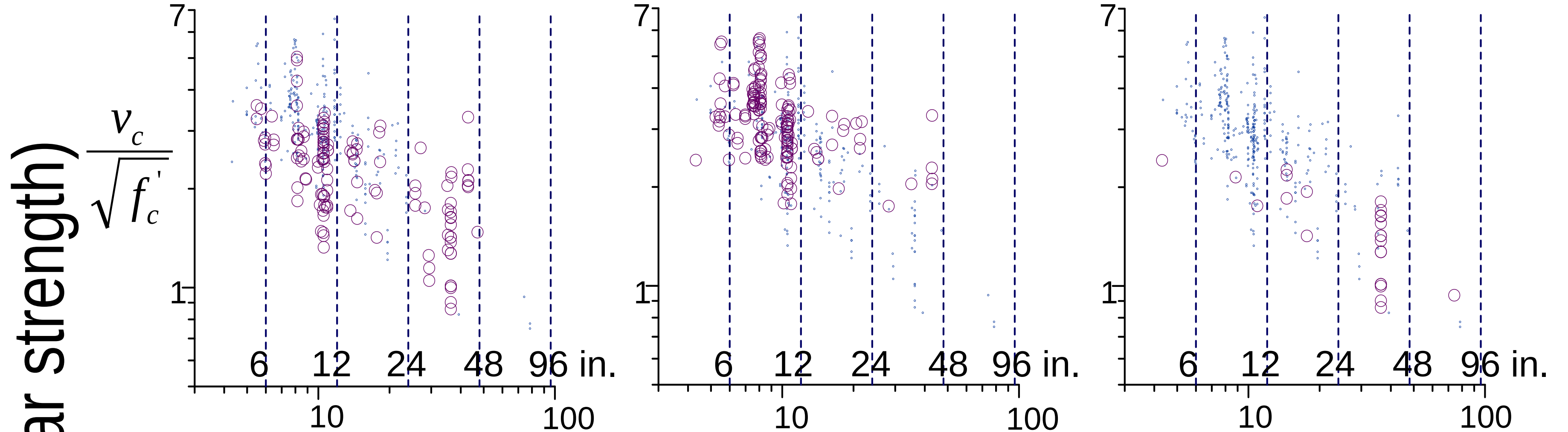
$$\overline{\rho}_w = 1.5\% \quad (\overline{a/d} = 3.3, \quad \overline{d}_a = 0.67 \text{ in.})$$

$$\overline{\rho}_w = 2.5\% \quad (\overline{a/d} = 3.3, \quad \overline{d}_a = 0.67 \text{ in.})$$

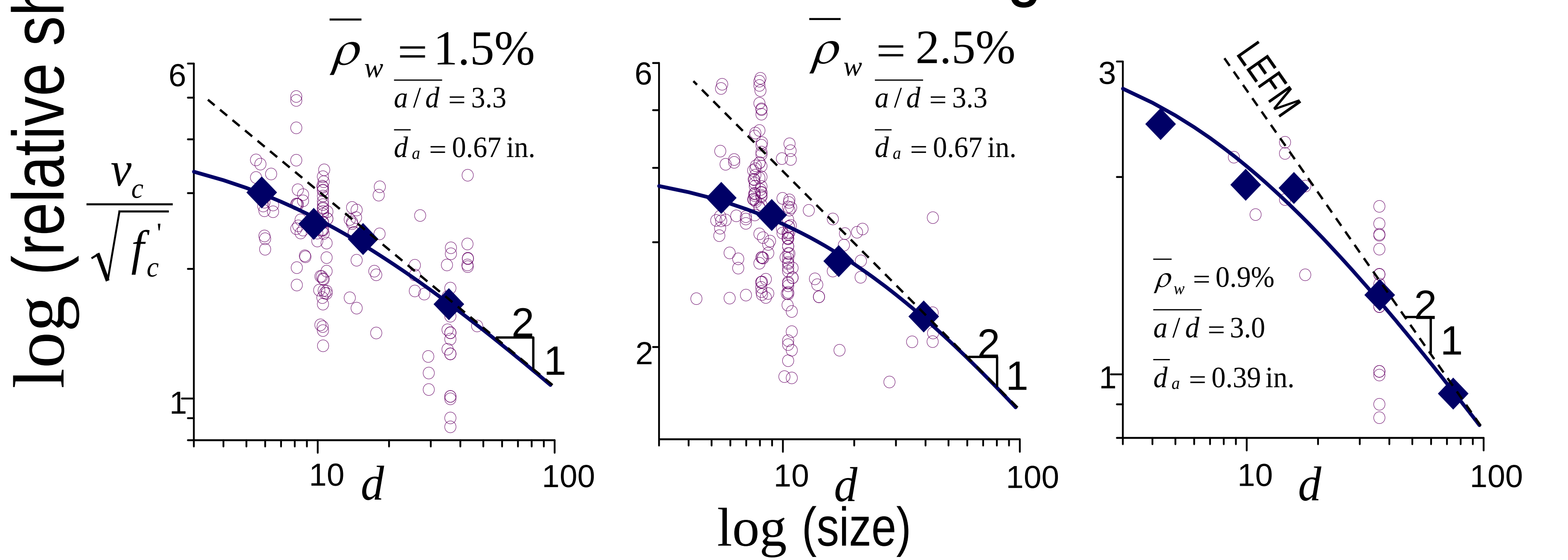


Circles remain, dots omitted

Restricting Strength Range of Data to Achieve Approx. the Same Means of ρ_w, a/d, d_a in All Size Intervals



Interval Centroids of Remaining Data Points



Size Effect Law Fitting Centroids of Filtered Database with Uniform $\overline{\rho}_w, \overline{a/d}, \overline{d}_a$

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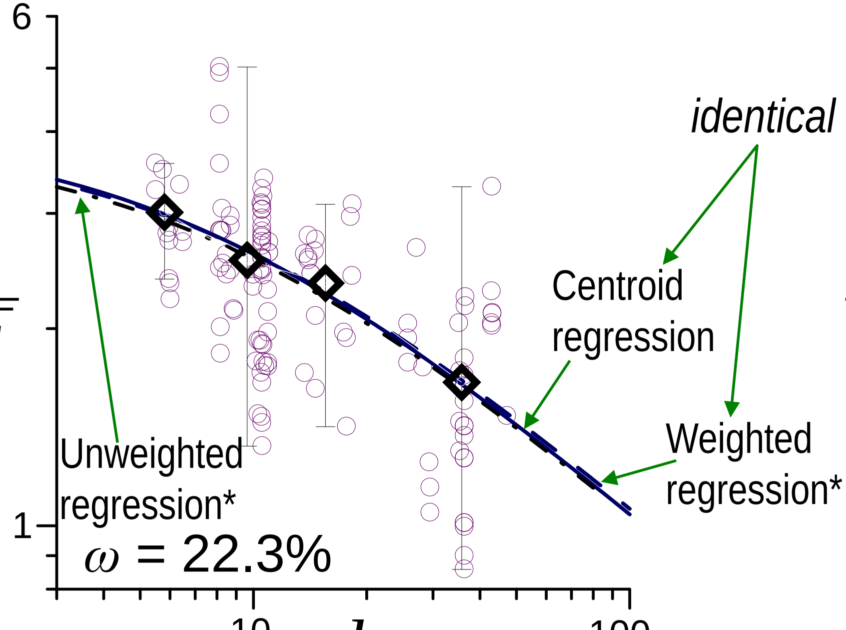
$\overline{a/d} = 3.3, \ \overline{d}_a = 0.67$ in.



$\overline{\rho}_w = 2.5\%$

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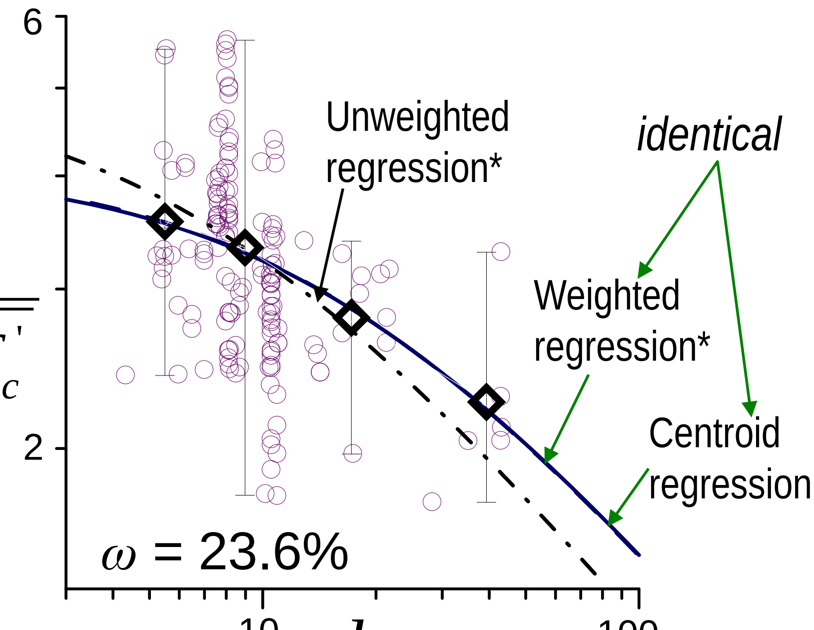


$\frac{v_c}{\sqrt{f_c'}}$ vs. d

log (size)

---

* of individual data points: (weight) ~ 1 / (number of points in interval)

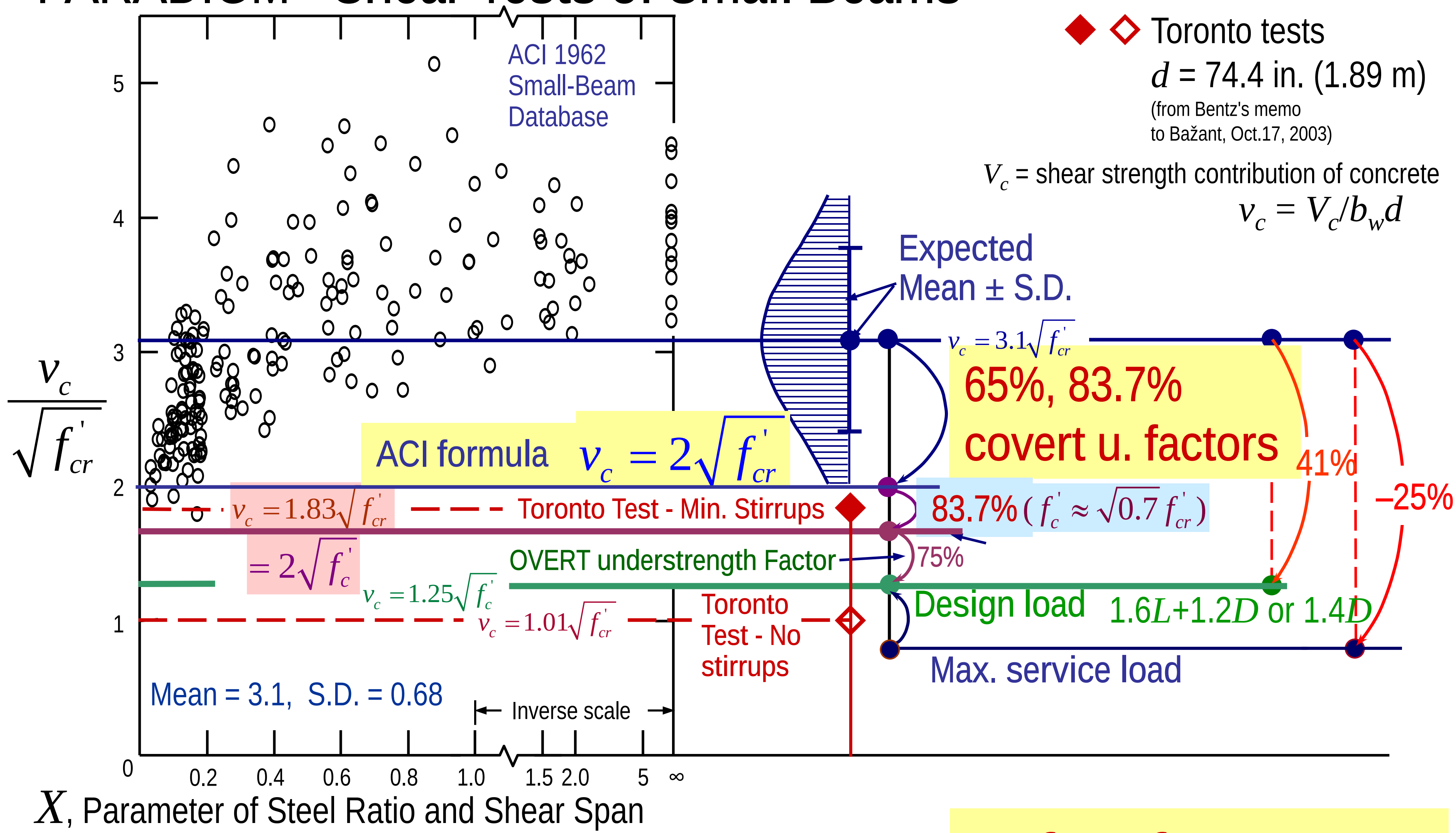
EFFECT OF STIRRUPS:

- **Stirrups can push the size effect up by an order of magnitude of D/l_0 but cannot prevent it.**

- **Increasing stirrup ratio in large beams is ineffective and can even reduce the shear strength.**

Covert Safety Factors

PARADIGM - Shear Tests of Small Beams



Safety factor ≈ 3.5

Overt and Covert Understrength Factors

Safe Design Criterion

Currently: $\text{Max}(1.6L + 1.2D,\ 1.4D;\ \ldots) \leq \varphi F$

φ = strength reduction (understrength) factor intended to distinguish **brittleness**

φ = 0.75 for shear

F = load capacity by design formula

Problem: F is a fringe formula, involving

Covert understrength factors:

- φ_f — for formula error
- φ_m — for material strength randomness

Required Revision:

$$\text{Max}(1.6L + 1.2D,\ 1.4D;\ \ldots) \leq \varphi\ \varphi_f \varphi_m F$$

$\varphi_f \approx 0.65$, $\varphi_m \approx 0.83$ for shear

Obscuring Effect of Covert Understrength Factors on Forensic Evidence

Safety factor for shear:

- Small beam:

$$\varphi = \frac{1.6}{0.75 \times 0.83 \times 0.65} = 3.8 \;\; = \text{average}$$

- range from 2.3 to 7

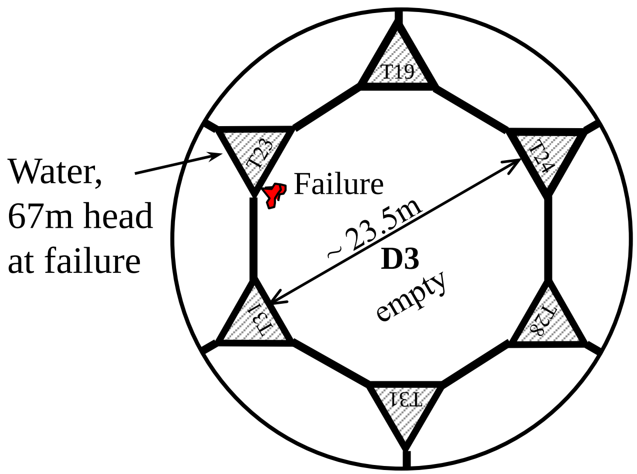
- Large beam (for size effect ratio = 2):

$$\varphi = \frac{1.4}{0.75 \times 0.83 \times 0.65 \times 2} = 1.7 \;\; = \text{average}$$

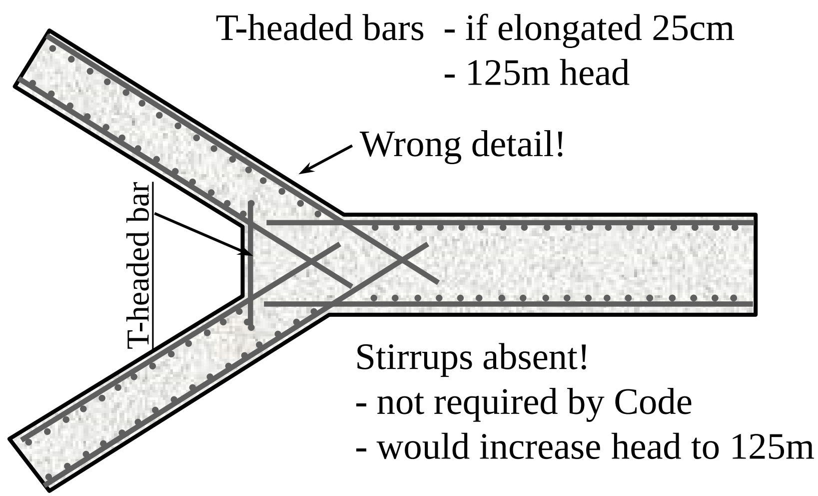
range from 1.05 to 3

➡ Why is the size effect rarely identified, in analyzing disasters? More than one mistake is needed to bring down a structure.

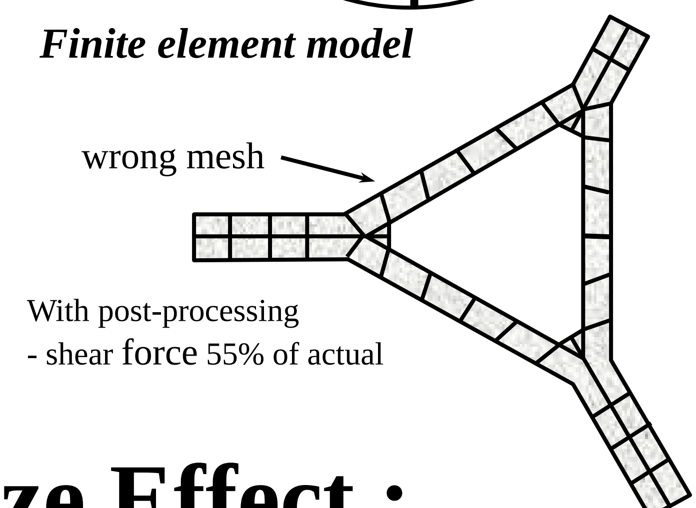
Failure of Sleipner A Platform

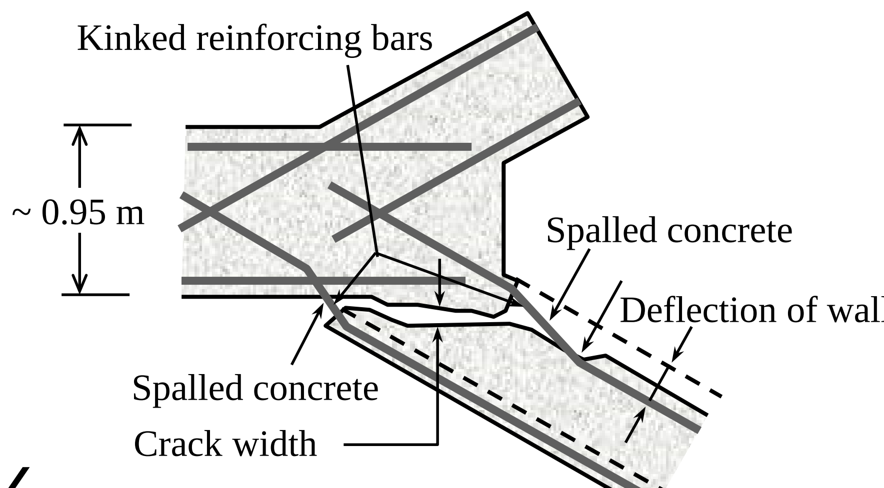


Finite element model



Assumed failure mode





Posterior p_{max} ~ 0.8 - 1.2 $p_{applied}$!

Size Effect :

f_t' reduced by 40%

How to Remedy Misleading Covert Understrength Factors in Codes

Option 1

- Use **mean** prediction formulas, **not fringe** formulas
- Use mean material strength $\overline{f}_c$ (or f'_{cr}), **not** reduced strength f'_c
- In addition to the current understrength factor $\varphi = \varphi_b$, accounting for brittleness, impose understrength factors :

– φ_f, for error of formula

– φ_m, for material strength randomness

and specify their C.o.V.'s ω

For shear: φ_b = **0.75,** φ_f = **0.65,** φ_m =**0.83**

Option 2

- Keep the current fringe formulas
- Keep the reduced strength f'_c
- Specify the implied understrength factors

φ_f and φ_m

with the coefficients of variations, e.g.

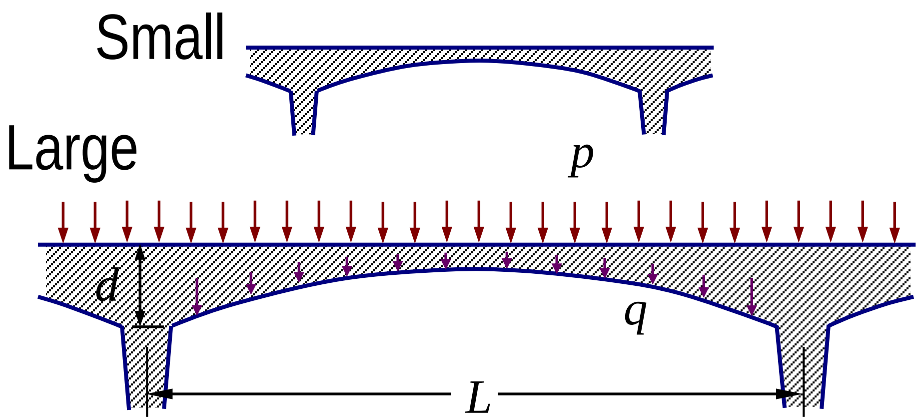
ω_f = 22% and ω_m = 19%

and with the probability cut-offs, e.g.

65% and 75%

- Only this will render reliability assessment meaningful !

Wrong Size Effect Hidden in Excessive Self Weight Factor



$$\sigma_N^{Code} = \text{Max}(1.4D,\ 1.2D + 1.6L)$$

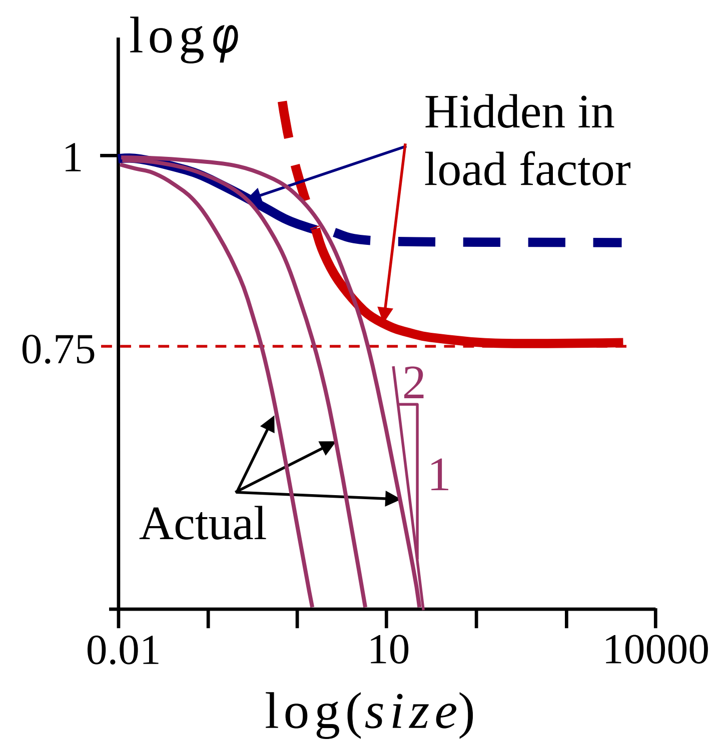
$$\sigma_N^{True} = 1.05D + 1.6L$$

Hidden size effect factor:

$$\varphi = \frac{\sigma_N^{Code}}{\sigma_N^{True}} = \frac{1.05C_1 d + 1.6C_2}{\text{Max}(1.4C_1 d,\ 1.2C_1 d + 1.6C_2)}$$

$$= 0.75 \text{ to } 1 \qquad (d = \text{size})$$

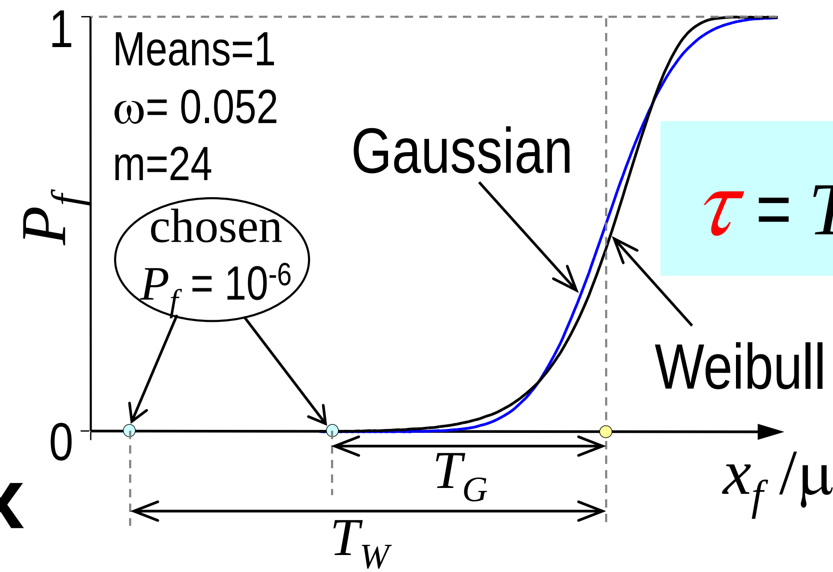
Wrong for: Flexural vs. shear failure, high-strength vs. normal concrete, prestressed vs. normal concrete, etc.



Revised Reliability Indices

a) **Cornell Index**

$$\beta = \frac{\mu_L - \mu_R}{\sqrt{s_L^2 + \tau^2 s_R^2}}$$



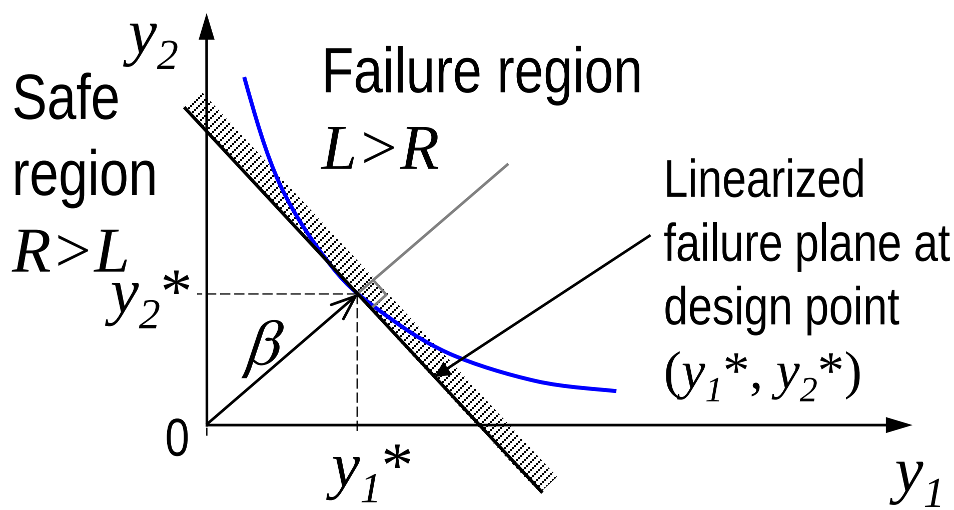
$\tau = T_W/T_G$

b) **Hasofer-Lind Index**

$$\beta = \min \sqrt{\sum_{i=1}^{n} y_i'^2}$$

$$y_i' = \frac{y_i - \mu_{y_i}}{\tau_i s_{y_i}}$$

τ_i = 1 for load
≥ 1 for resistance



Neither FORM nor SORM but EVRM (extreme value RM)

Safe Design Criterion

Currently: $$\text{Max}(1.6L + 1.2D,\ 1.4D;\ \ldots) \le \varphi F$$

φ = strength reduction (understrength) factor intended to distinguish **brittleness**

φ = 0.75 for shear

F = load capacity by design formula

Problem: F is a fringe formula, involving

Covert understrength factors:

- φ_f — for formula error
- φ_m — for material strength randomness

Required Revision:

$$\text{Max}(1.6L + 1.2D,\ 1.4D;\ \ldots) \le \varphi\ \varphi_f \varphi_m F$$

$\varphi_f \approx 0.65$, $\varphi_m \approx 0.83$ for shear

Size Effect in Punching Shear

II. Size Effect in Punching Shear of RC Slabs

ACI-445C-Database for Punching of Slabs

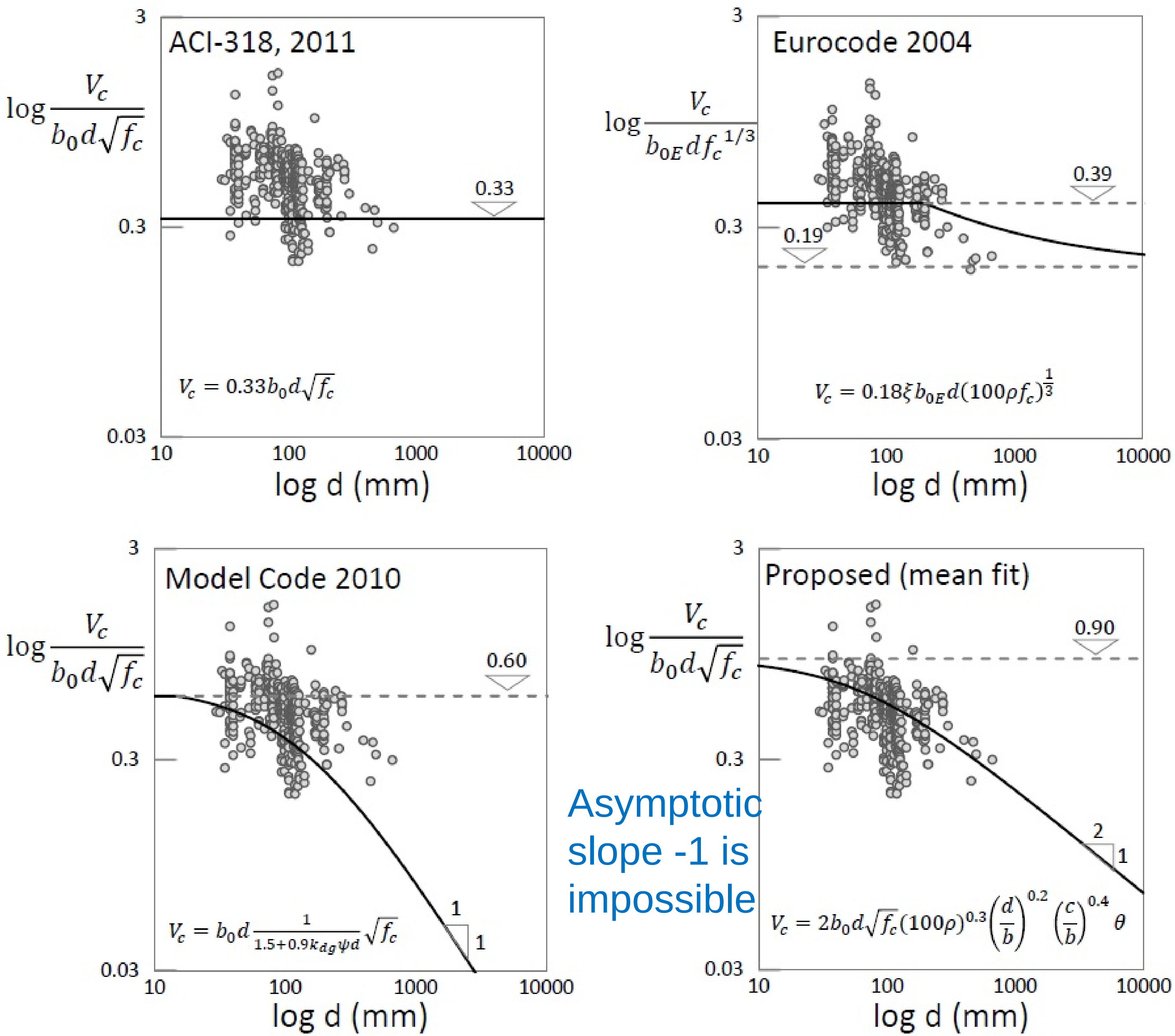
440 Tests (60 researchers)

d (effective depth of slabs) ranges from 1.18 in. to 26.3 in (from 30 to 668 mm).

f_c (concrete strength) ranges from 1160 to 17114 psi (from 8 to 118 MPa)

ρ (longit. reinforcement ratio) ranges from 0.1% to 7.3%

Formulae in Main Design Codes



Asymptotic slope -1 is impossible

V_c=total capacity
b_0=control perimeter
b_{0E}=control perimeter (EC)
d =effective depth
f_c =concrete strength
ξ =Size effect term in EC
ρ =reinforcement ratio (longitudinal)
ψ =slab rotation term (MC)
θ =size effect term in proposed model

Database Filtering

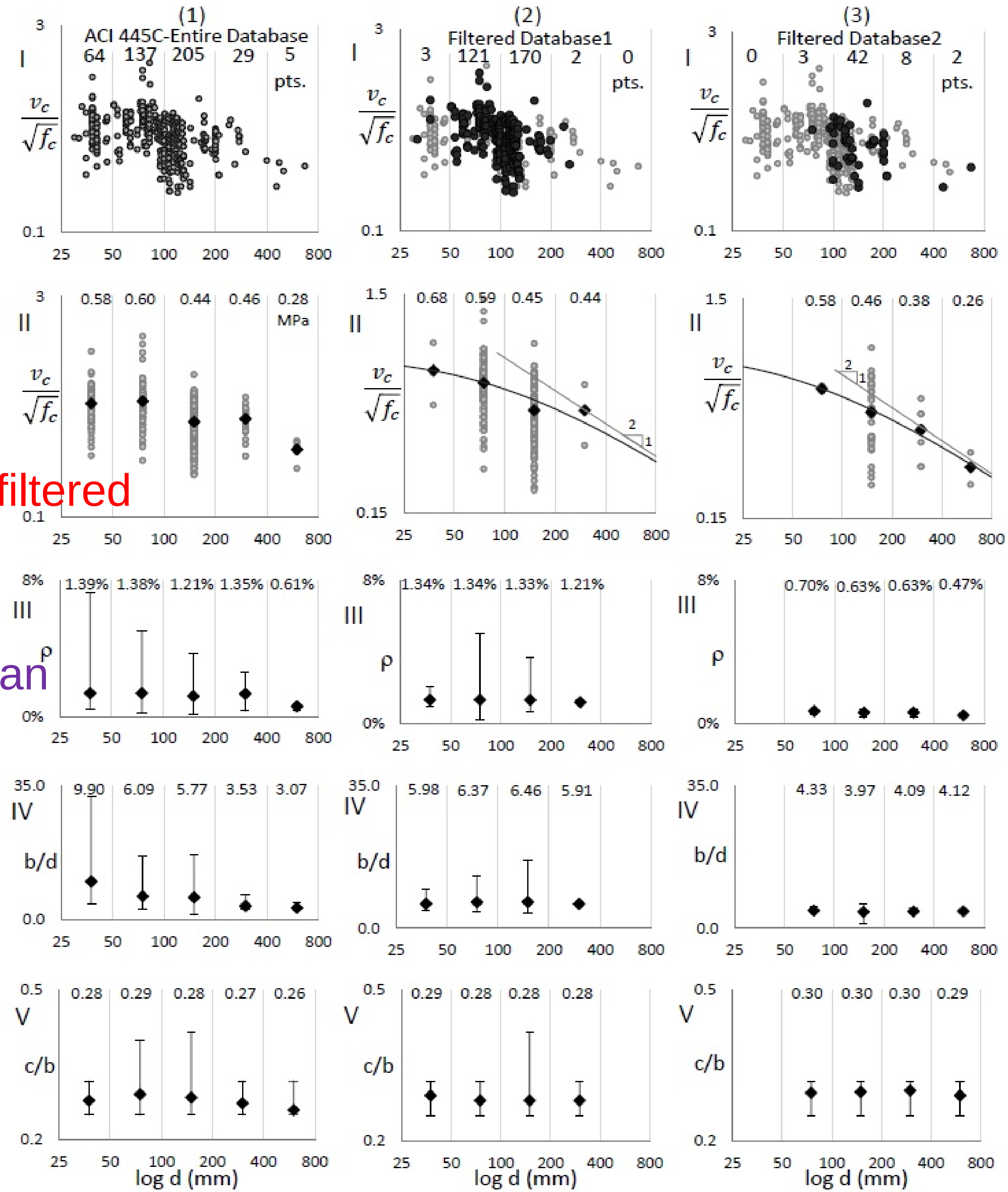
to reveal the size effect trend <u>without any calculations</u>

Unfiltered

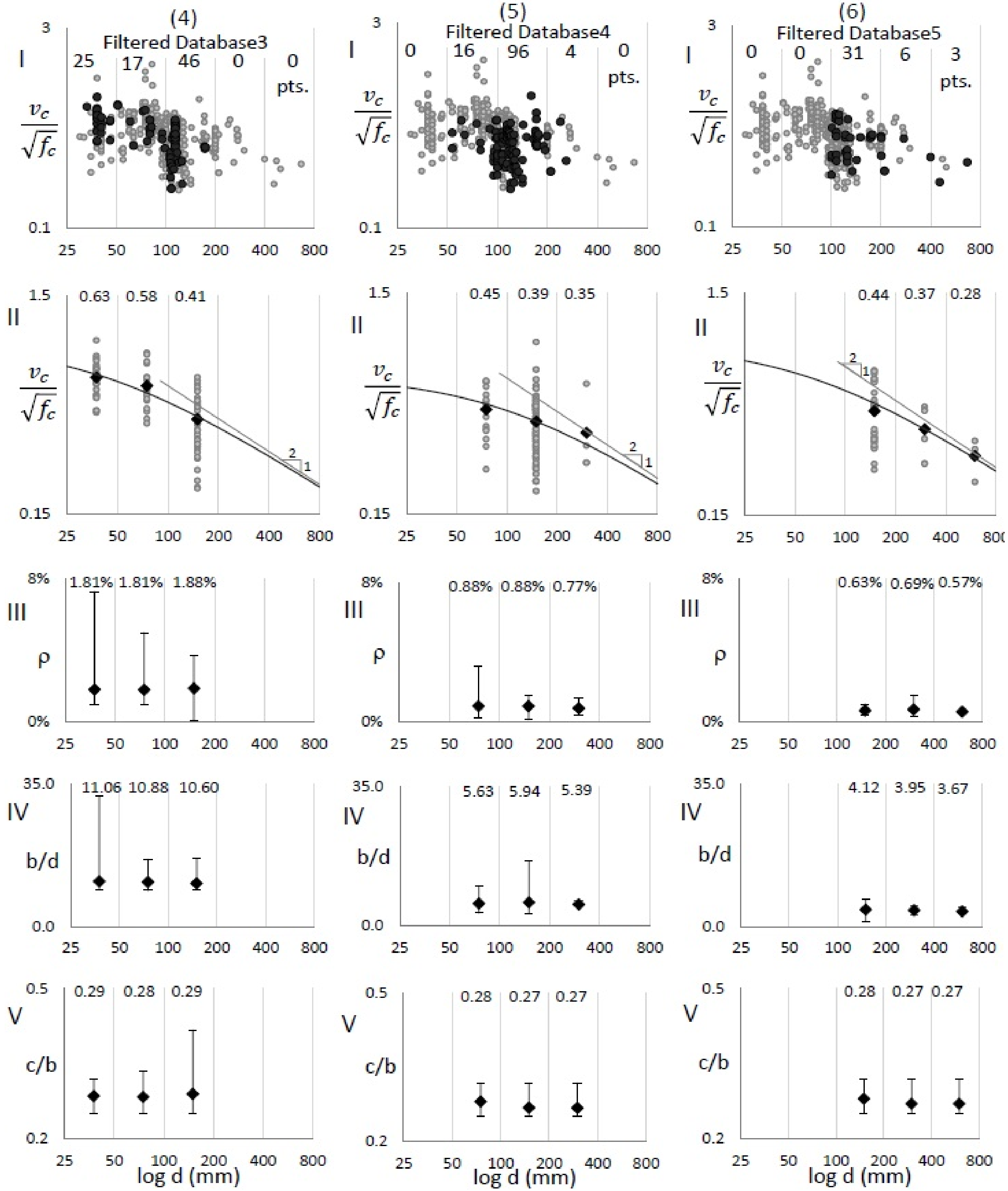
Filtered mean steel ratio

Filtered mean *b/d*

Filtered mean *c/b*



Database filtering (continued)



Mean steel ratios

Mean *b/d*

Mean *c/b*

Premise:

1) The tests relevant to size effect are limited and not scaled

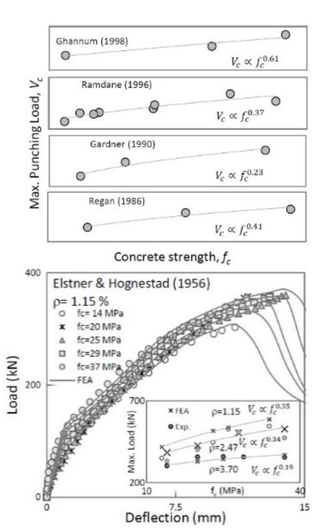
2) Microplane model M7 gives generally excellent fits of test data on concrete failures

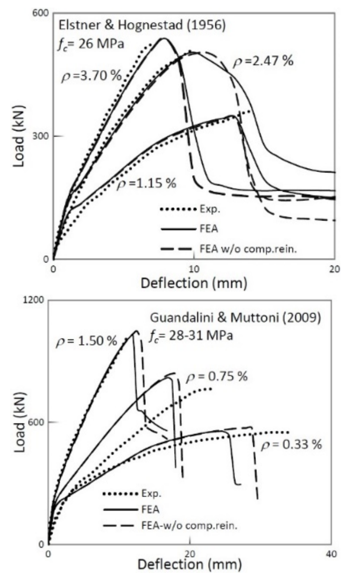
Approach:

- Calibrate M7 parameters by fitting existing test data with limited size range and different structural geometries
- Then use calibrated M7 to predict the size effect by simulating scaled specimens

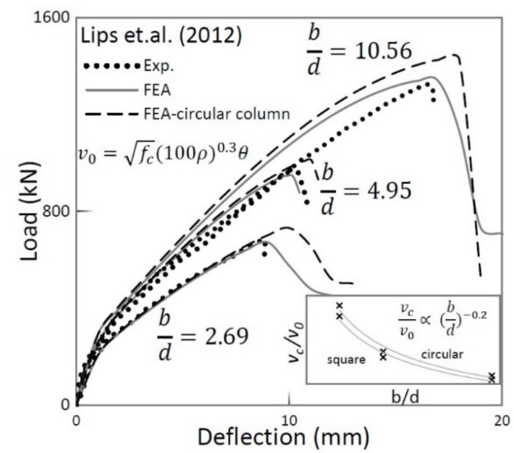
Calibration of M7 by Multivariate Regression of Test Data for Effect of Other Parameters

Tests by Elstner and Hognestad 1956





Tests by Lips et al. 2012

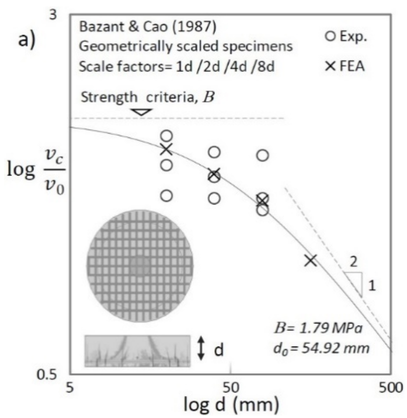


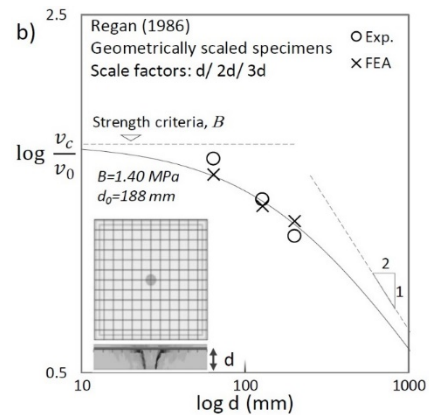
Verification with microplane model M7 for slabs without shear reinforcement

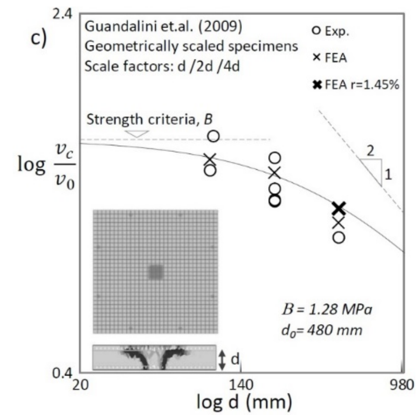
Tests by Bazant and Cao, 1987

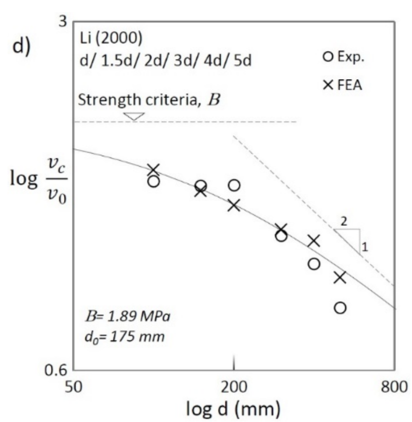
Tests by Regan, 1986

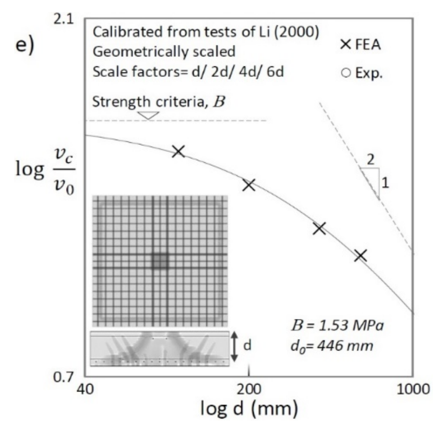
Tests by Guandalini and Muttoni, 2009











Tests by Li, 2000

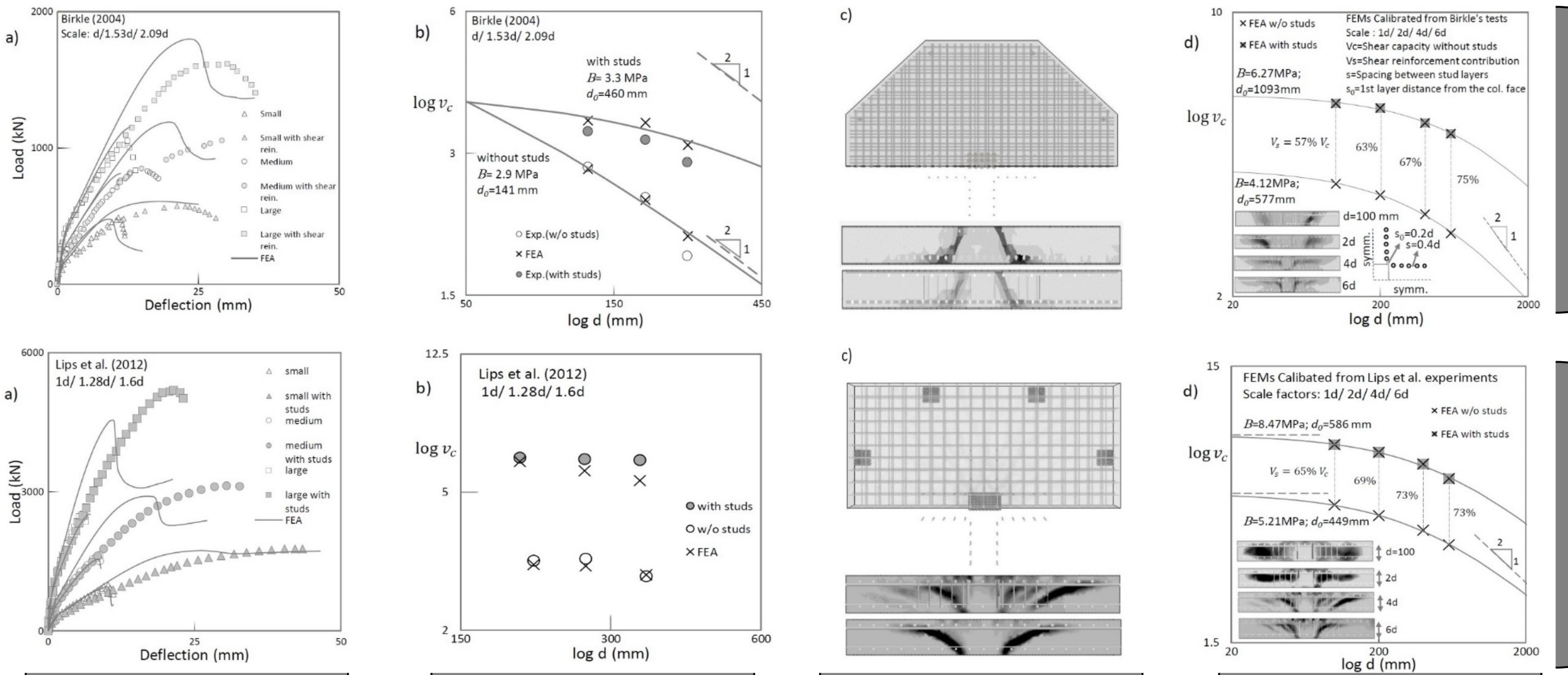
Cross points hold for finite element analysis results (simulations of the experimental works)

Circle points hold for the experimental results

Solid lines refer to the size effect fit of the FEM results

Verification with microplane model, for slabs with shear reinforcement

Test data of Birkle, 2004, and fits by proposed size effect equaton



Tests of Birkle

Tests of Lips et al.

Load-Deflection curves of tests and corresponding FEMs

Not perfectly scaled specimens (1 : 1.6)

Finite element models and corresponding fracture patterns

Perfectly scaled FEM without and with shear reinforcement

Proposed model and calibration by multivariate regression

$$V_u = V_c + V_s$$

$$V_c = b_0 d v_c$$

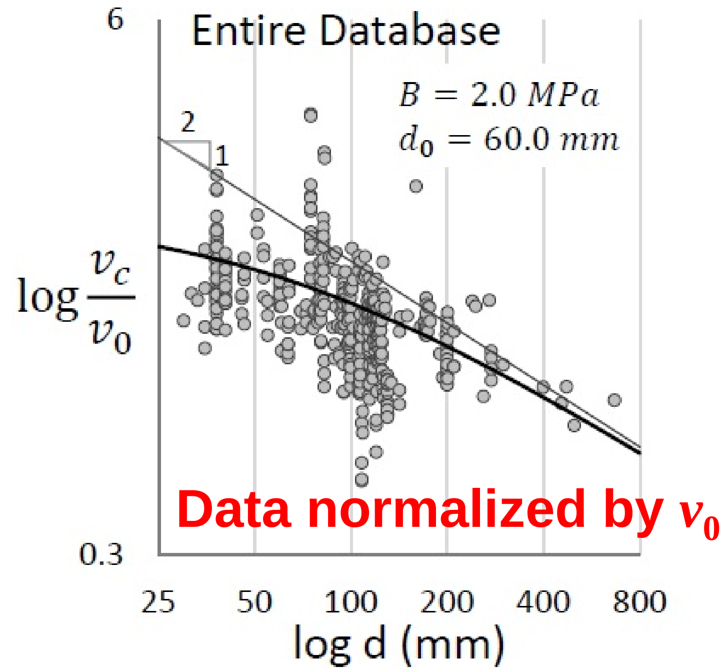
$$v_c = v_0 \theta$$

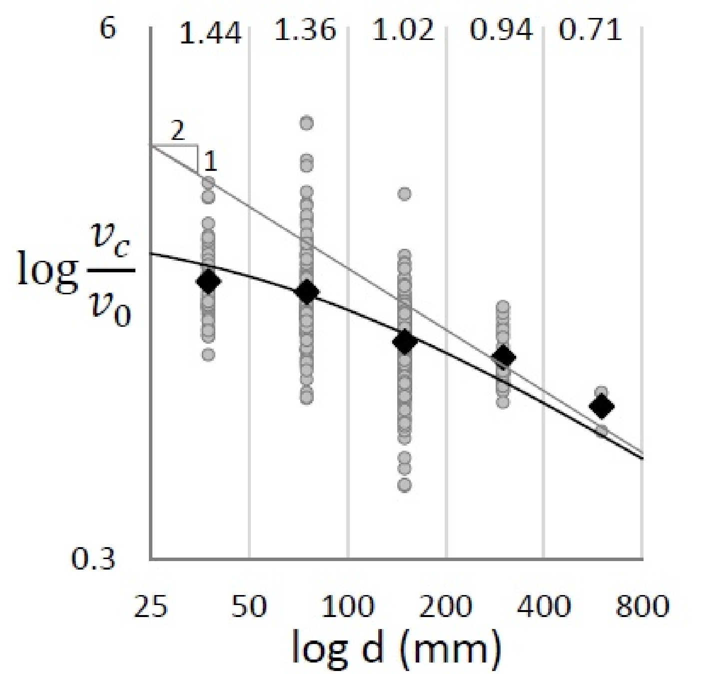
$$v_0 = \lambda \sqrt{f_c} (100\rho)^{0.3} \left(\frac{d}{b}\right)^{0.2} \left(\frac{c}{b}\right)^{0.4}$$

$$\theta = \frac{1}{\sqrt{1 + d/d_0}}$$

$$V_s = \lambda_s A_{sw} f_{yw} \left(\frac{d}{s_w}\right) \left(\frac{c}{b}\right)^{0.3}$$

D = slab thickness, c = side of column, b = its perimeter





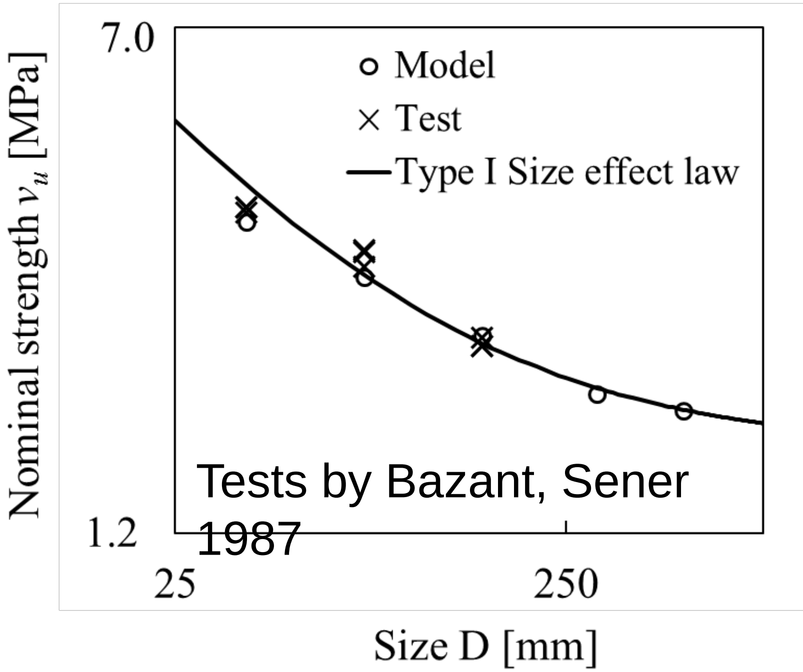
Size Effect in Torsion

Type I !

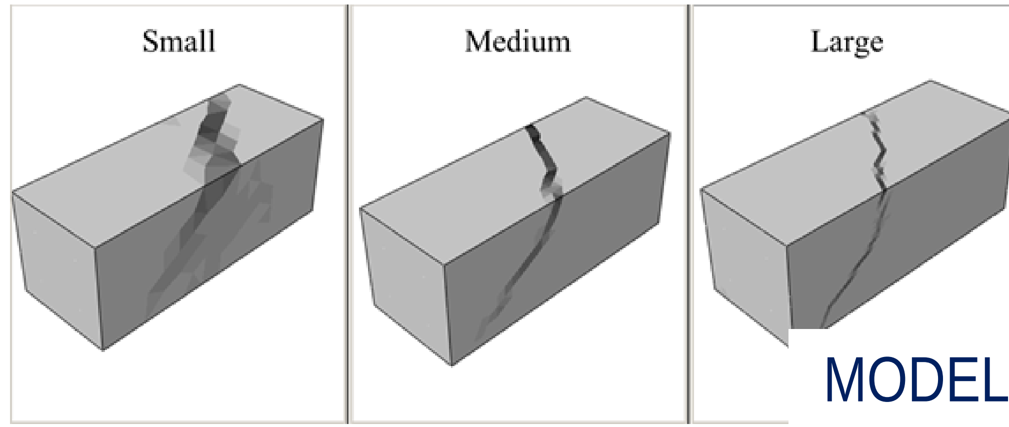
II. Size Effect in Torsional Failures

Plain concrete beams, solid cross sections

Size effect plot (log-scale)

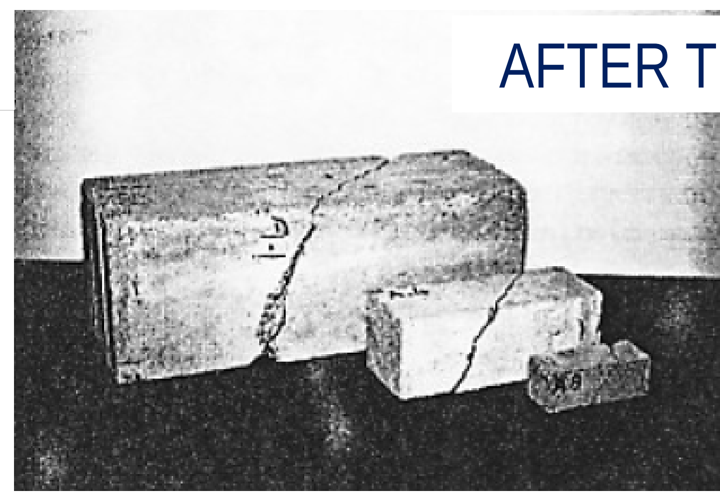


Failure pattern



MODEL

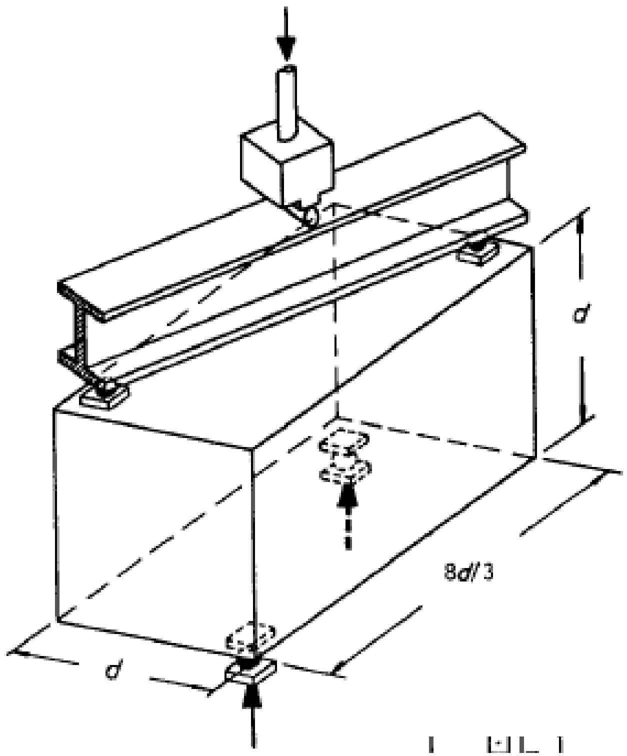
AFTER TEST



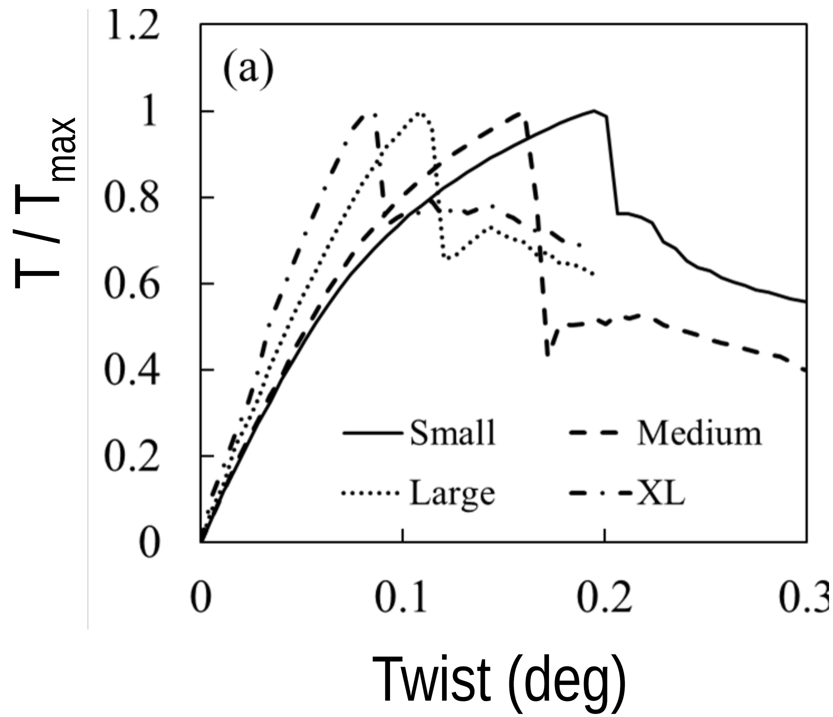
Bažant, Z. P., and Sener, S., "Size Effect in Torsional Failure of Concrete Beams," Journal of Structural Engineering, V. 113, No. 10, 1987, pp. 2125-2136.

Size Effect on Torsional Failure

Plain concrete beams – reduced scale tests



Scaled computed torque-twist curves

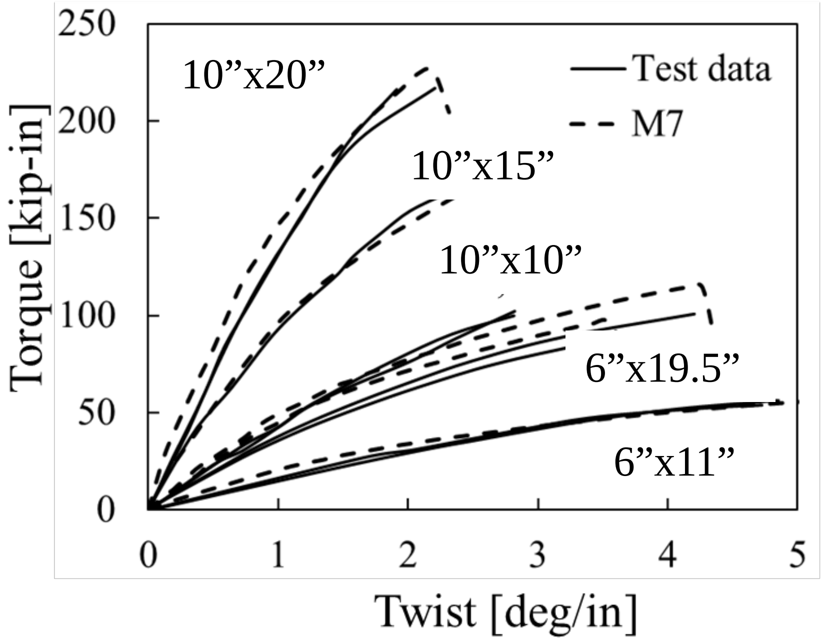


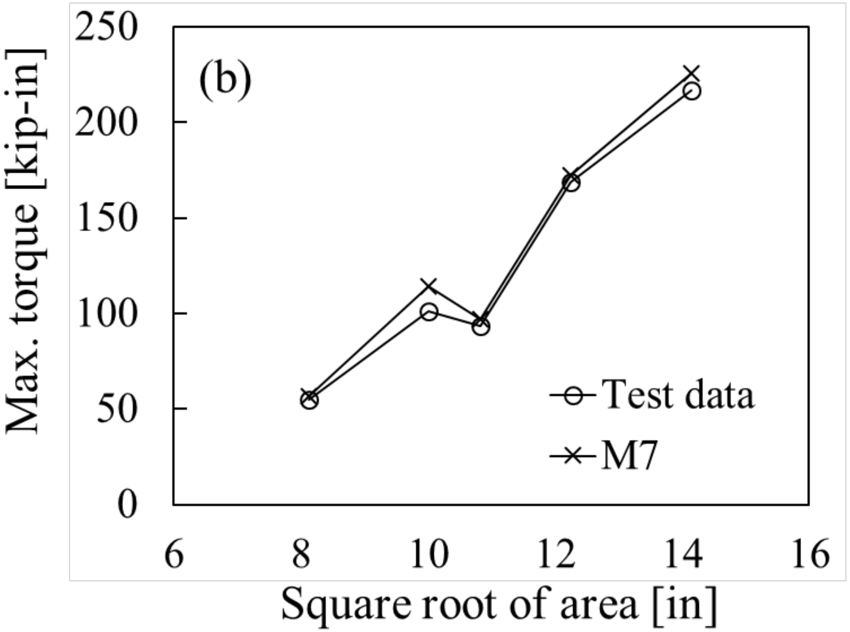
- 3 sizes tested in lab, *d* = 1.5”, 3”, 6”)
- Size range extended using microplane model M7, to 1:13
- Type I failure – as soon as peak torque is reached

Bažant, Z. P., and Sener, S., “Size Effect in Torsional Failure of Concrete Beams,” Journal of Structural Engineering, V. 113, No. 10, 1987, pp. 2125-2136.

Torsional failures : model validation

Plain concrete beams tested by Hsu (1968)

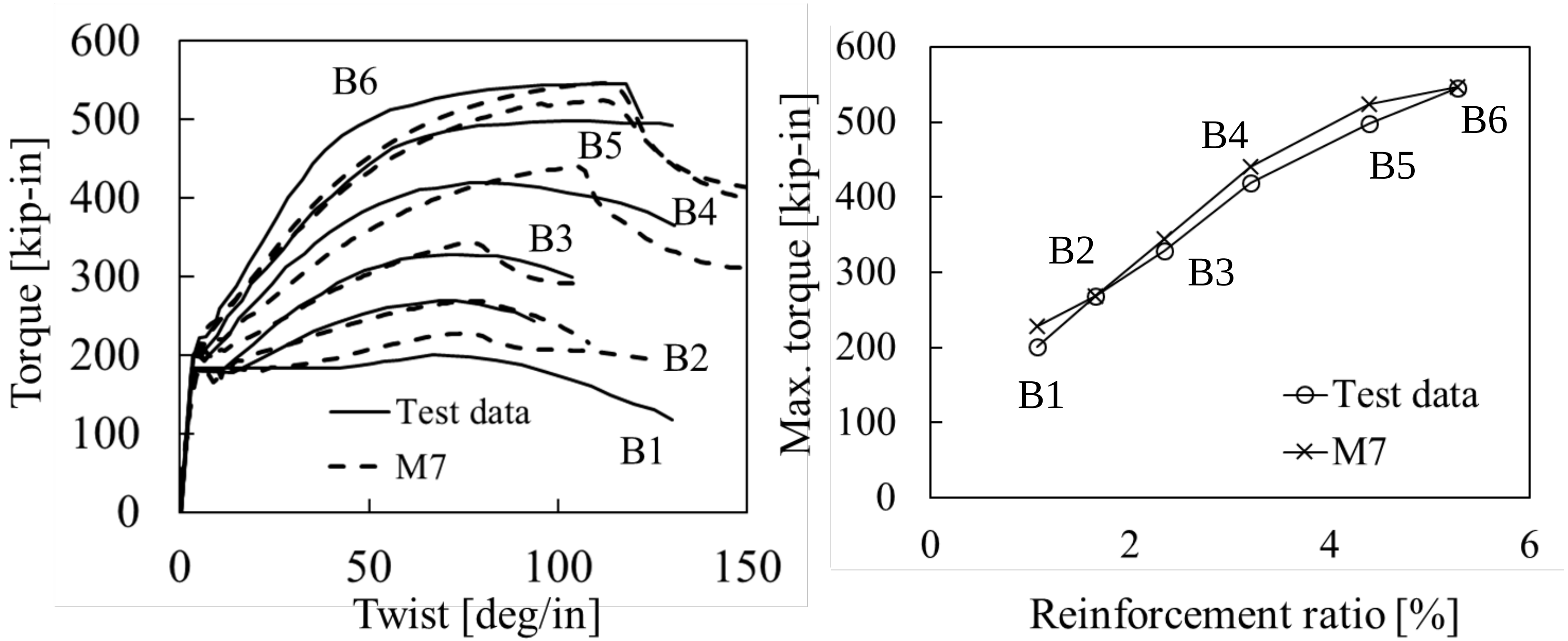




- Effects of size (and shape) observed in tests
- Model predictions match tests very well – lends confidence to the model

Hsu, T. T. C., "Torsion of Structural Concrete—Plain Concrete Rectangular Sections," Torsion of Structural Concrete, SP-18, American Concrete Institute, Farmington Hills, MI, Jan. 1968, pp. 203-238.

Torsion of RC Beams: Model Validation



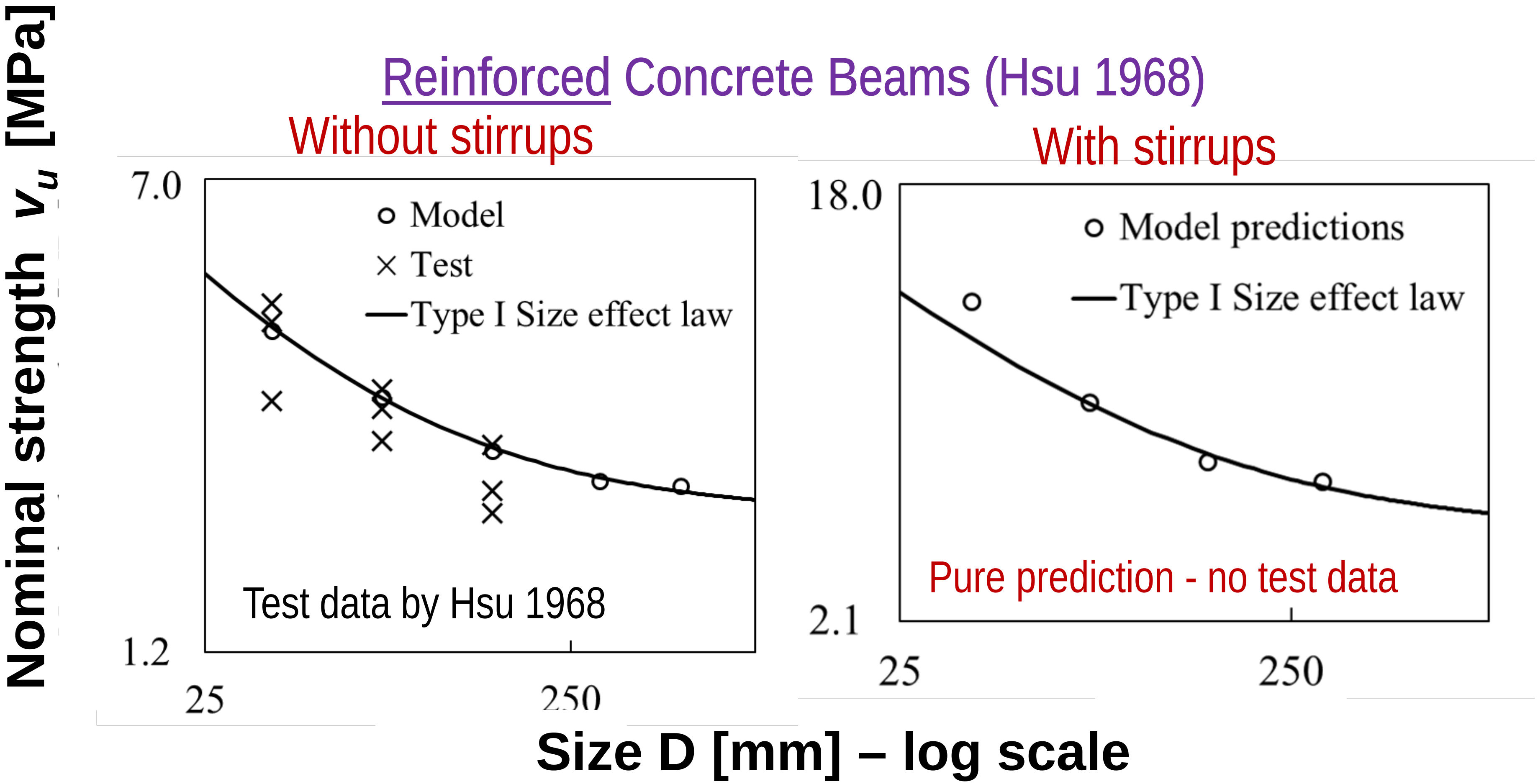
- Note: Strong effect of reinforcement ratio.
- Predictions by calibrated model match tests very well – lends confidence to the model

Hsu, T. T. C., "Torsion of Structural Concrete—Behavior of Reinforced Concrete Rectangular Members," Torsion of Structural Concrete, SP-18, Am. Concrete Institute, Farmington Hills, MI,

Size effect in torsional failuress

Interestingly, is of Type 1, even with stirrups

Reinforced Concrete Beams (Hsu 1968)



Kirane, K., Darun Singh, and Bažant, Z. P. "Size Effect In The Torsional Strength Of Plain And Reinforced Concrete", submitted to ACI Structural J.

Size Effect on Columns, Prestressed Beam Flexure, Composite Beams and Other Types of Failure

See Bazant's website

Code Articles Requiring Size Effect

- shear of beams without and with stirrups
- torsion of beams
- punching of slabs
- shear of deep beams
- bar splices and development length
- all failures due to compression crushing of concrete, as in
  -- columns,
  -- prestressed beams,
  -- arches
  -- bearing strength
  -- strut-and-tie models
- failure of composite beams due to failure of connections
- precast concr. connections
  -- grouted joints,
  -- shear keys,
  -- connectors,
  -- toppings
- softening seismic failures
- delamination of bonded laminate retrofit
- flexure of plain concrete
- strength reduction factors for brittle failures
- load factors for self-weight

Thanks for listening

Questions, please?